Modelling passengers' arrivals at public transport stops

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Abstract

The paper presents statistical exploration of passengers' arrivals at bus stops in urban public transport. First, it describes the methodology which is applied on the urban public transport system, where the following conditions are met: (i) passengers are familiar with the timetables, (ii) the vehicles run on time, and (iii) the capacity of the vehicles is sufficient. The methodology is demonstrated on the urban public transport in Žilina, Slovak Republic. The correlation analysis of the survey performed at transportation stops in Žilina reveals that there is a correlation between the waiting time and headway. The relationship between this variables can be modelled by a linear function or better by a logarithmic function. The Kolmogorov-Smirnov and the chi-square tests accept the hypothesis that the Gumbel minimum distribution is the suitable mathematical model of passengers' arrivals rates. The proposed models are necessary in the operations research and simulation methods used for public transport planning.

Keywords: public transport, passengers' waiting time, correlation analysis, regression analysis.

1. Introduction

The paper describes the results of our research aimed at the behaviour of passengers in urban public transport, particularly at their arrivals at bus stops. The research goal was:

1. to determine whether there exists a relationship between passengers' waiting time and the line frequency (headway between the successive buses);
2. if the waiting time depends on the headway, to determine the mathematical model of this dependence;
3. to propose a suitable mathematical model of passengers' arrivals rate at a stop.

The time passengers spend at the stops waiting for a bus has been regarded as an important criterion of the public transport system quality (see for example Osuna and Newell, 1972; Daganzo, 1997; Avineri, 2004; Fan and Machemehl, 2004; Álvarez et al., 2010). Therefore, it is important to know the aspects affecting passengers' arrivals at the stops in order to design an effective public transport system or to improve its quality. Moreover, description of arrival patterns by mathematical models is necessary if one wants to use sophisticated methods for public transport planning, such as operations

research methods (Fan and Machemehl, 2004, Jánošíková et al., 2012, Janáček and Koháni, 2010) or a computer simulation (Erath et al., 2013).

There is a widespread belief that the waiting time depends on the line frequency. That is why in several studies regarding public transport quality only frequency is considered (e.g. Hensher et al., 2003; Eboli and Mazzulla, 2008a, 2008b). Published mathematical models of waiting time consider both the headway and the arrival time (or waiting time) as random variables. The most simple model is based on the assumption that passengers do not know the timetables, they arrive to the original stop randomly, and the mean waiting time is proportional to the headway (inversely proportional to the line frequency, respectively). Under the assumption that passengers arrive at a constant rate, the waiting time is a function of the mean headway and its variance:

$$E(W) = \frac{E(H)}{2}\left(1 + \frac{D(H)}{E^2(H)}\right),$$ (1)

where $E(W)$ denotes the mean waiting time, $E(H)$ the mean headway and $D(H)$ the headway variance.

When a transportation service operates with long headways, passengers do not arrive at stops randomly but they tend to arrive few minutes before the planned vehicle departure. Previous studies performed in Europe in the 1970s (for the survey, see Luethi et al., 2006) and in the U.S.A. (Fan and Machemehl, 2004) were aimed at the determination of the minimum headway with non-random arrival pattern and the model for the relationship between the waiting time and the headway. The headway threshold varied from 5 to 12 minutes and the models were linear or quadratic. The recent European study was carried out in Zürich by Luethi et al. (2006). Passengers were supposed to belong to one of two groups: those who were familiar with the schedule and those who did not know the schedule. As a consequence, the authors suggest an arrival rate model that combines the uniform distribution for informed passengers with the shifted Johnson S_B distribution for uninformed passengers. The Johnson distribution is shifted with a small value due to the observation that some passengers arrive short time after the vehicle departure. The reported share of these passengers is quite high (from 5 to 16 %). The authors explain passengers' early arrivals by the fact that passengers do not trust the service reliability and rely on a regular delay but then they do not catch the bus. Passengers waiting the whole period would be observed also if they failed to board due to the insufficient vehicle capacity. Regarding the dependence of the mean waiting time on the headway, Luethi et al. propose the logarithmic model.

2. Methodology

The first goal of our study was to find out whether the waiting time depends on the headway even in the case of reliable service with the sufficient capacity of vehicles and with passengers being familiar with the schedules. We proceeded from the situation in the Slovak Republic, which is similar to most European countries, where the public transport users are well-informed. The timetables are available at the stops as well as on the Internet, so passengers are able to obtain schedule information almost everywhere by using new information technologies.

As it was said before, both the waiting time (or passengers' arrival time, respectively) and the headway are random variables. The dependence of the random variables is the matter of the *correlation analysis*. The measure of the dependence between two random variables X and Y is a so called correlation coefficient $\rho(X,Y)$. It is obtained by dividing the covariance of the two variables by the product of their standard deviations. The correlation coefficient ranges from -1 to 1. The value of 0 implies that there is no linear correlation between the variables. If $\rho(X,Y) \neq 0$, then X and Y are dependent. The value of 1 implies that the linear equation describes the relationship between X and Y perfectly, with all data points lying on the line for which Y increases as X increases. The value of -1 implies that all data points lie on the line for which Y decreases as X increases.

To find $\rho(X,Y)$ we need to know the probability distributions of the variables. However, the probability distributions are unknown in practice. We usually have observed or measured only a realization of the sample, i.e. n values (x_i, y_i). These values can be used to estimate $\rho(X,Y)$ by the Pearson correlation coefficient:

$$R_{X,Y} = \frac{\sum_{i=1}^{n} x_i y_i - n\overline{x}\overline{y}}{\sqrt{\sum_{i=1}^{n} x_i^2 - n\overline{x}^2}\sqrt{\sum_{i=1}^{n} y_i^2 - n\overline{y}^2}} , \tag{2}$$

where $\overline{x}$ and $\overline{y}$ are arithmetic means of the values x_i and y_i, respectively.

Because some properties of this estimate depend on the probability distribution of the pair (X, Y), another estimate of $\rho(X,Y)$ was designed, which does not depend on the joint distribution of (X, Y). It is called the Spearman correlation coefficient, and it is based on the ranks of values x_i and y_i. The values x_i and y_i are sorted in a non-decreasing order: $x_1 \leq x_2 \leq ... \leq x_n$ and $y_1 \leq y_2 \leq ... \leq y_n$, respectively. Let us denote the ranks of the values x_i and y_i by r_i and q_i, respectively. The similarity between ranks r_i and q_i shows the relationship between the values x_i and y_i. The Spearman correlation coefficient is given by the following term:

$$R_S = 1 - \frac{6}{n(n^2-1)}\sum_{i=1}^{n}(r_i - q_i)^2 \tag{3}$$

The estimated correlation coefficients $R_{X,Y}$ and R_S are almost always different from zero. Therefore a statistical test should be preformed to verify whether their value is statistically significant. The null hypothesis

H_0: $\rho = 0$ (correlation is insignificant)

is tested against the alternative hypothesis

H: $\rho \neq 0$.

Several tests with different test criteria are available. The test criteria are functions of the estimated correlation coefficient.

The first test is based on the assumption that the sample of the pairs (x_i, y_i) for $i = 1, ..., n$ comes from the two-dimensional normal distribution with the correlation coefficient ρ. The test criterion

$$T = \frac{R_{X,Y}\sqrt{n-2}}{\sqrt{1-R_{X,Y}^2}} \tag{4}$$

has the Student's t distribution with $n - 2$ degrees of freedom under the null hypothesis.

The second test is based on the same assumption as the normal distribution. The test uses the Fisher z transformation that converts the Pearson correlation coefficient to the variable Z. The formula for the transformation is:

$$Z = \frac{1}{2}\ln\frac{1+R_{X,Y}}{1-R_{X,Y}} \tag{5}$$

Z is approximately normally distributed with the mean $\frac{1}{2}\ln\frac{1+\rho}{1-\rho}$ and the standard deviation $\frac{1}{\sqrt{n-3}}$.

The test criterion

$$Z^* = \frac{\sqrt{n-3}}{2}\ln\frac{1+R_{X,Y}}{1-R_{X,Y}} \tag{6}$$

has the standard normal distribution $N(0,1)$ under the null hypothesis.

If the assumption of the above mentioned tests is not met, the nonparametric test for the Spearman correlation coefficient can be used. The value R_s is compared with the tabulated critical value r_α. If $|R_S| \le r_\alpha$, then the null hypothesis is accepted.

Further, the confidence interval of the correlation coefficient can be calculated using Fisher Z variable. The $100(1-\alpha)$ % confidence interval of the theoretical correlation coefficient ρ is defined as follows:

$$\rho \in \left\langle \mathrm{tgh}\left(Z - \frac{z_\alpha}{\sqrt{n-3}}\right), \mathrm{tgh}\left(Z + \frac{z_\alpha}{\sqrt{n-3}}\right)\right\rangle, \tag{7}$$

where z_α is the value of the inverse distribution function of the standard normal distribution $N(0,1)$ at the point $\frac{2-\alpha}{2}$, and the function $\mathrm{tgh}(x)$ is the hyperbolic tangent:

$$\mathrm{tgh}(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

In the case that random variables X and Y are dependent, one can describe their relationship by a *regression function*. The most simple form of the regression function is the linear function $y = ax + b$, where a and b are unknown coefficients that need to be estimated. The most common method of estimation is the least squares method. Using this method we get the following estimates:

$$\widehat{a} = \frac{n\sum_{i=1}^{n}x_i y_i - \sum_{i=1}^{n}x_i\sum_{i=1}^{n}y_i}{n\sum_{i=1}^{n}x_i^2 - \left(\sum_{i=1}^{n}x_i\right)^2} \tag{8}$$

$$\widehat{b} = \frac{\sum_{i=1}^{n}y_i\sum_{i=1}^{n}x_i^2 - \sum_{i=1}^{n}x_i\sum_{i=1}^{n}x_i y_i}{n\sum_{i=1}^{n}x_i^2 - \left(\sum_{i=1}^{n}x_i\right)^2} \tag{9}$$

The point estimates (8) and (9) should be contrasted with confidence intervals. These can be constructed under the following assumptions:

1. values y_i are independent;
2. for every x_i the values y_i have the normal distribution $N(\mu,\sigma)$ with mean $\mu = ax_i + b$ and variance σ^2 identical for all x_i.

Then estimates $\widehat{a}$ and $\widehat{b}$ have also the normal distribution with the following characteristics:

$$E(\hat{a}) = a, D(\hat{a}) = \frac{\sigma^2}{nS_X^2} \tag{10}$$

$$E(\hat{b}) = b, D(\hat{b}) = \frac{\sigma^2}{n^2 S_X^2} \sum_{i=1}^{n} x_i^2 \tag{11}$$

where

$$S_X^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2 \tag{12}$$

The variance σ^2 is not known and it must be estimated by

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} \left(y_i - (\hat{a}x_i + \hat{b}) \right)^2 \tag{13}$$

The corresponding 100(1-α) % confidence intervals are then as follows:

$$a \in \left\langle \hat{a} - \frac{\hat{\sigma}}{S_X \sqrt{n-2}} t_\alpha^{n-2}, \hat{a} + \frac{\hat{\sigma}}{S_X \sqrt{n-2}} t_\alpha^{n-2} \right\rangle \tag{14}$$

$$b \in \left\langle \hat{b} - \frac{\hat{\sigma}}{S_X} \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n(n-2)}} t_\alpha^{n-2}, \hat{b} + \frac{\hat{\sigma}}{S_X} \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n(n-2)}} t_\alpha^{n-2} \right\rangle \tag{15}$$

where t_α^{n-2} is the critical value of the Student t distribution with $n - 2$ degrees of freedom.

Using these confidence intervals we can test the hypothesis about nonzero values of the coefficient. It means that regarding the coefficient a, the null hypothesis

H_0: $a = 0$ (the coefficient is insignificant)

is tested against the alternative hypothesis

H: $a \neq 0$.

The null hypothesis is accepted if the confidence interval contains 0, i.e.

$$0 \in \left\langle \hat{a} - \frac{\hat{\sigma}}{S_X \sqrt{n-2}} t_\alpha^{n-2}, \hat{a} + \frac{\hat{\sigma}}{S_X \sqrt{n-2}} t_\alpha^{n-2} \right\rangle.$$

A similar test can be performed for the coefficient b of the regression function.

The variability of the observed values y_i can be measured through different sums of squares:

$$S_y = \sum_{i=1}^{n} (y_i - \bar{y})^2 \tag{16}$$

$$S_{reg} = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2 \tag{17}$$

$$S_{err} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \tag{18}$$

where $\hat{y}_i = \hat{a}x_i + \hat{b}$ is the value on the regression line. S_y is the total sum of squares of deviations of the measured values y_i from their mean, S_{reg} is the regression sum of squares, also called the explained fraction of variance, and S_{err} is the residual sum of squares, also called the unexplained fraction of variance. It holds $S_y = S_{reg} + S_{err}$. The ratio

$$R^2 = \frac{S_{reg}}{S_y} \qquad\qquad (19)$$

is a so called coefficient of determination. R^2 is a statistics that will give some information about the goodness of fit of a model. It takes values between 0 an 1. The better the linear regression fits the data in comparison to the simple average, the closer the value of R^2 is to one. An R^2 of 1 indicates that the regression line fits the data perfectly.

In addition to the R^2 statistics, the model validation can be done through the F-test on the statistical significance of the regression model.

Let us denote $SS_{reg} = \dfrac{S_{reg}}{p-1}$ and $SS_{err} = \dfrac{S_{err}}{n-p}$, where p is the number of regression parameters. The regression model is considered to be statistically significant if SS_{reg} is significantly greater than SS_{err}. The null hypothesis

H_0: $SS_{reg} = SS_{err}$ (the regression model is insignificant)

is tested against the alternative hypothesis

H: $SS_{reg} > SS_{err}$.

The test criterion

$$F = \frac{SS_{reg}}{SS_{err}} \qquad\qquad (20)$$

has the Fisher-Snedecor distribution with the parameters $v = p - 1$ and $w = n - p$ under the null hypothesis.

The approach derived for the linear regression model can be applied for some non-linear models as well, for example for the logarithmic function $y = a\ln(x) + b$. The method for the parameter estimation is the same; the only exception is that the logarithm $\ln(x_i)$ is used instead of x_i.

3. Case study

As a case study for passengers' arrivals and waiting time modelling we chose the urban public transport in the city of Žilina. Žilina is a middle-sized city situated in the north-western part of the Slovak Republic. It has 81,494 inhabitants (by 2011), and covers the area of 80 km^2.

The transportation service in the city is provided by the transportation operator Dopravný podnik mesta Žiliny, s.r.o. (DPMŽ). During the day, 8 trolleybus lines and 10 bus lines operate. At night, the city area is served by 1 bus line.

The data for the analysis were collected at 6 stops in Žilina on weekdays during the morning peak and off-peak periods (from 6:00 to 11:00). The stops were selected according to the following criteria:

- Passengers are not supposed to change lines at the stop.
- The stop must be busy enough to enable collecting sufficient data.

The data were collected "by hand", i.e. by observing passengers' arrivals at stops and recording the passenger's arrival time, the number of the line taken by the passenger, and the vehicle departure time.

3.1 The results of the correlation analysis

Using the measured data we want to determine whether there exists a relationship between passengers' waiting time (random variable Y) and the line headway (random variable X). The size of the sample used in the following calculations is $n = 467$.

The relationship is measured by the Pearson and Spearman correlation coefficients that were computed using the terms (2) and (3), and take the values $R_{X,Y} = 0.134$ and $R_S = 0.759$. Both coefficients are different from zero, which means that the waiting time and the headway are related by the monotonic function. This finding can be verified by the test of significance of the theoretical correlation coefficient ρ. The null hypothesis

H_0: $\rho = 0$ (correlation is insignificant)

is tested against the alternative hypothesis

H: $\rho \neq 0$.

at the level of significance $\alpha = 0.05$.

In the first two tests with the Pearson correlation coefficient it is assumed that the sample of pairs (X,Y) comes from the normal distribution. As regards the variable X (the line headway), it takes only several values, most often 10, 15, 20, and 30 minutes that are common in public transport operation and therefore it is impossible to make a test on its probability distribution. The sample of Y was tested on the probability distribution for particular headways x_i (see Section 3.3). Although the chi-square test failed to reject the hypothesis about the normal distribution for some x_i, for all data together the hypothesis was rejected at the significance level $\alpha = 0.05$. Although the assumption of the first two tests was not proved, all three tests mentioned in Section 2 were performed.

The value of the test criterion according to (4) is $T = 2.919$. It is greater than the critical value of the Student distribution for the significance level 0.05 and n-2 degrees of freedom (1.965), therefore we reject the null hypothesis H_0 and accept the alternative hypothesis H that X and Y are dependent random variables.

The value of the test criterion according to (6) is $Z^* = 2.907$. It is greater than the critical value of the standard normal distribution for the significance level 0.05 (1.96), therefore we reject the null hypothesis H_0 and accept the alternative hypothesis H that X and Y are dependent random variables.

The same outcome is obtained using the third nonparametric test with the Spearman correlation coefficient. The value $R_s = 0.759$ is greater than the tabulated critical value $r_\alpha = 0.091$, so the null hypothesis is rejected.

Further, the confidence interval of the Pearson correlation coefficient can be calculated. According to (7), the 95% confidence interval is $\langle 0.044, 0.222 \rangle$. This interval includes the Pearson correlation coefficient $R_{X,Y} = 0.134$. This fact confirms the correlation between the waiting time and the headway.

3.2 The results of the regression analysis

To specify the dependence mathematically, a regression function can be derived, which describes the dependence of the pair of random variables (X,Y). Using the least square method, a linear and a logarithmic functions were proposed, further the significance of coefficients was investigated and the quality of both models was examined using the F-test.

The linear function was estimated as $y = 0.088x + 3.932$. The 95% confidence intervals of the coefficients are: $a \in \langle 0.029, 0.147 \rangle$, $b \in \langle 2.770, 5.095 \rangle$. None of the

intervals contains 0, therefore both coefficients a and b are significant. The F-test states the null hypothesis

H_0: $SS_{reg} = SS_{err}$ (the regression model is insignificant)

against the alternative hypothesis

H: $SS_{reg} > SS_{err}$.

The value of the test criterion according to (20) is $F = 8.521$. It is greater than the critical value of the Fisher-Snedecor distribution with the parameters $v = 1$ and $w = 465$ for the significance level 0.05 ($F_{0.95}(1,465) = 3.861$), therefore we reject the null hypothesis H_0 and accept the alternative hypothesis H that the explained fraction of variance is significantly greater than the unexplained fraction of variance. The coefficient of determination for this linear model is $R^2 = 0.018$.

The logarithmic function was estimated as $y = 1.930\ln(x) + 0.047$. The 95% confidence intervals of the coefficients are: $a \in \langle 0.923, 2.937 \rangle$, $b \in \langle -2.857, 2.950 \rangle$. The confidence interval for coefficient b contains 0, but this only means that the absolute part of the function may be zero, however the type of the function is still logarithmic.

The value of the F-test criterion according to (20) is $F = 14.185$. It is greater than the critical value of the Fisher-Snedecor distribution ($F_{0.95}(1,465) = 3.861$), therefore we reject the null hypothesis H_0 and accept the alternative hypothesis H that the logarithmic model is statistically significant. The coefficient of determination is $R^2 = 0.030$.

The coefficients of determination for both regression models are quite small. It means that the values $\widehat{y}_i$ on the regression curve are far away from the observed values y_i. The reason is that for each headway x_i there were a lot of different waiting times y_i observed, which can also be seen in Figures 1 and 2. In accordance with Luethi et al. (2006), the logarithmic dependence seems to be a better approximation of the relationship between the examined random variables.
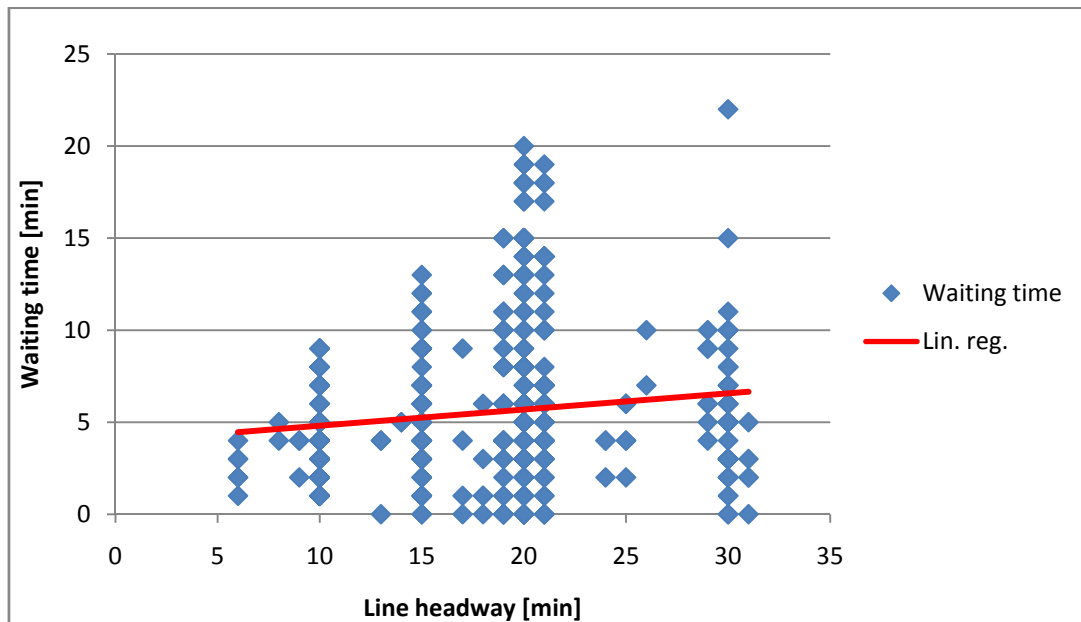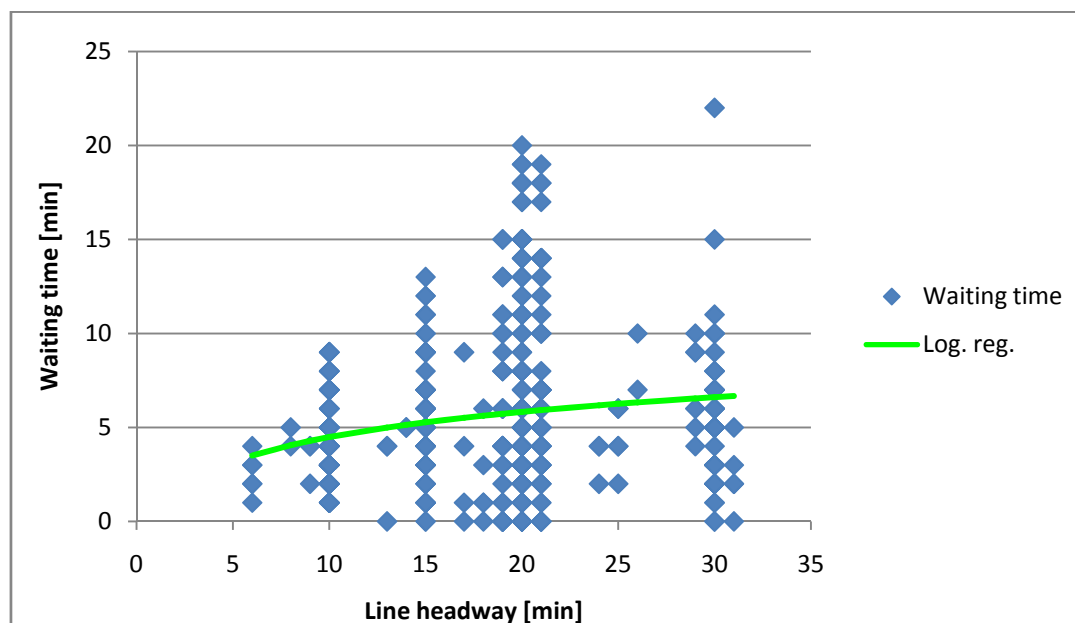

Figure 1. Linear regression

Figure 2. Logarithmic regression

3.3 The distribution of the arrival rates

The next step in our research was to specify the distribution of passengers' arrivals for the most common headways (10, 15, 20, and 30 minutes). The random variable is now the time elapsed between the departure of the previous vehicle and the arrival of the passenger. The frequency diagrams of the passengers' arrival times were constructed for each headway. The diagrams suggested that the Gumbel minimum distribution would be a suitable model. The formula for the probability density function (PDF) of the Gumbel minimum distribution is

$$f(x) = \frac{1}{b} \exp\left(\frac{x-a}{b} - \exp\left(\frac{x-a}{b} \right) \right) \tag{21}$$

for $x \in (-\infty, \infty)$; $a \in (-\infty, \infty)$ is the location parameter and $b > 0$ is the scale parameter.

Using the Kolmogorov-Smirnov and the chi-square tests we can accept the hypothesis that the arrivals follow the Gumbel minimum distribution. Moreover, for the 15-minute headway also the normal distribution was accepted by the tests.

For illustration, Figures 3 to 6 display the relative frequencies of passengers' arrivals and the PDF of the Gumbel minimum distribution for four most common headways. As it can be seen, the location parameter of the PDF strongly depends on the headway, since passengers tend to arrive at the boarding stop few minutes before the planned bus departure. Only a couple of passengers arrive at the beginning of the period. So we can conclude that most passengers are familiar with the timetables and adjust their arrivals to the schedule.
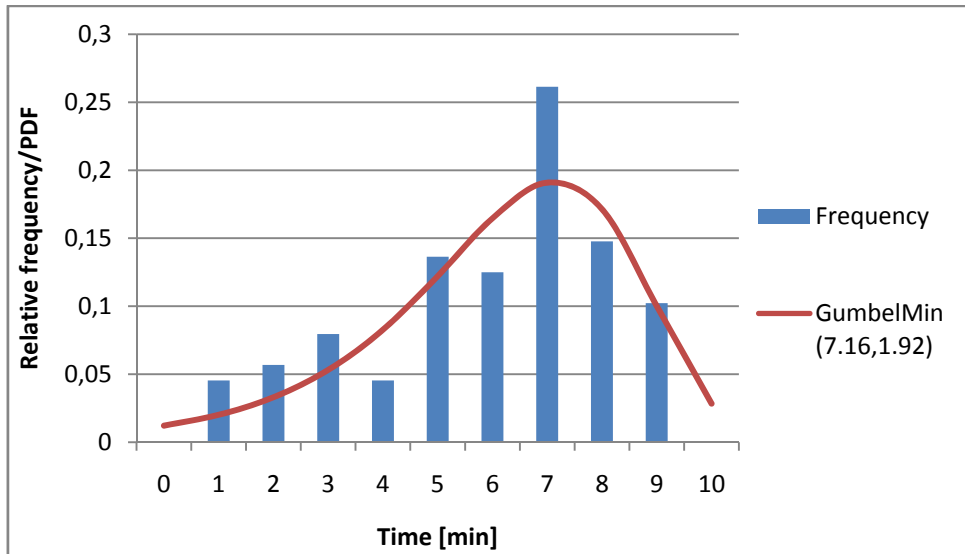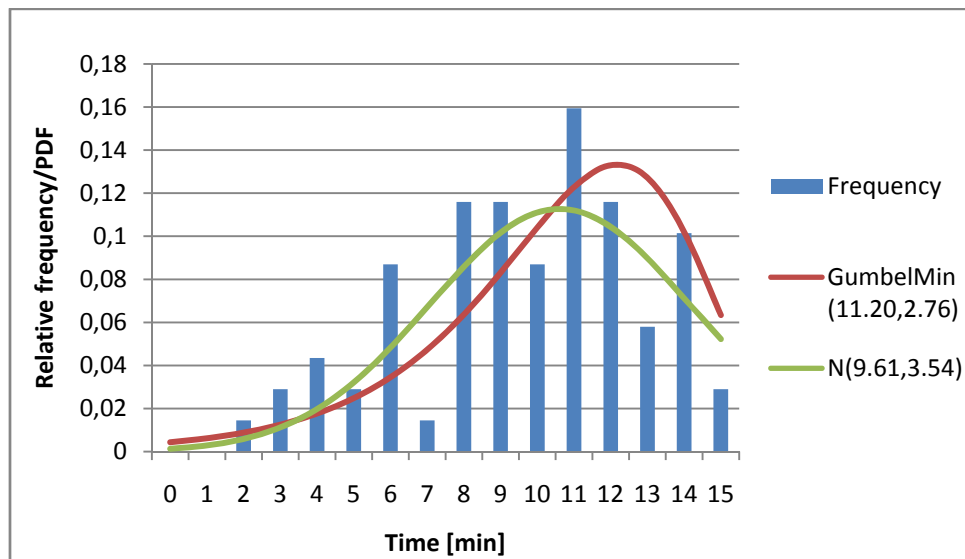
Figure 3. Passengers' arrivals in 10-minute headway

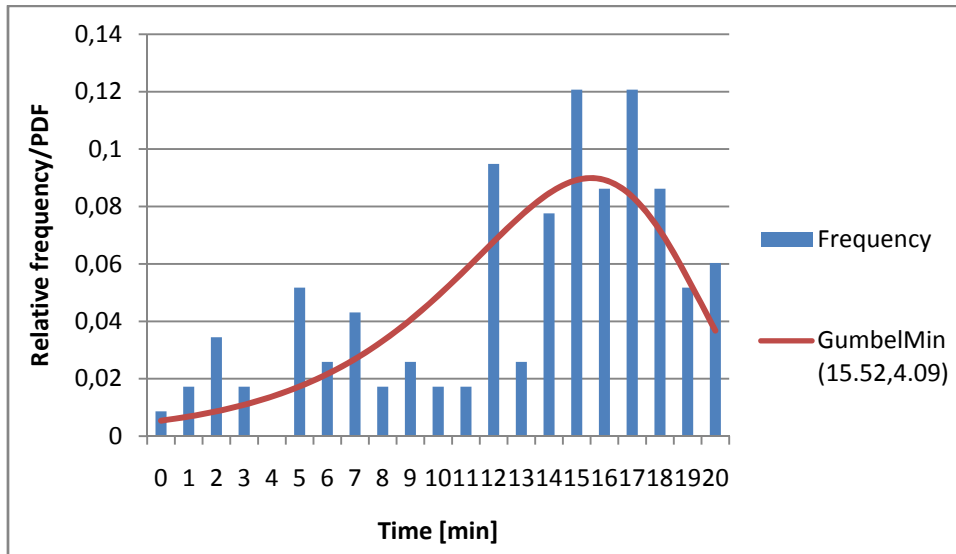

Figure 4. Passengers' arrivals in 15-minute headway

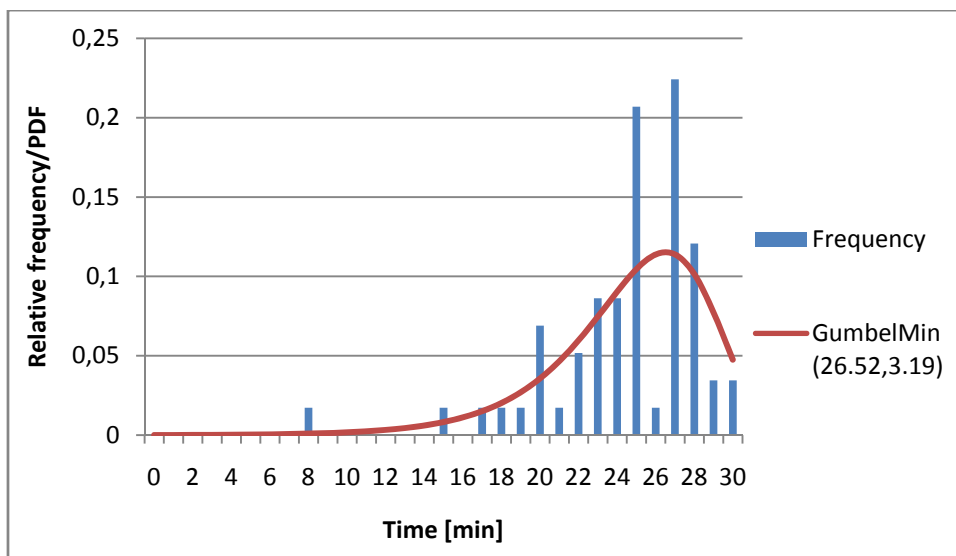*Figure 5*. Passengers' arrivals in 20-minute headway

Figure 6. Passengers' arrivals in 30-minute headway

4. Conclusions

We proposed a methodology where statistical methods are used for the exploration of passengers' arrivals at bus stops in urban public transport. The main results of our research are as follow:

1. Passengers' waiting time and line headway are correlated random variables.
2. The relationship between this variables can be modelled by a linear function or better by a logarithmic function.

3. The Gumbel minimum distribution is the suitable mathematical model of passengers' arrivals; the location parameter of the PDF strongly depends on the line headway.

The results can be generalized for every public transport system in which the users are well-informed. The proposed models are necessary in the operations research and simulation methods used for improving public transport quality.

Acknowledgements

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