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Intra-City Call-Taxi Fleet Sizing using Petri Net Embedded Simulation Optimization

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Abstract

Call-taxi has seen tremendous growth during the past two decades and the smartphone app based call-taxi hiring for intra-city commuting has not only provided greater visibility to customers but also increased their expectations. Fleet sizing is an important decision to a call-taxi aggregator that strikes a balance between customer service in terms of call-taxi availability and the utilization of the fleet for a day's forecasted demand. An optimization via simulation approach is developed for optimizing call-taxi fleet size. The simulation model embeds a Petri Net model of a call-taxi system where the customer requests and travel times are stochastic. Three optimization models are used to determine the fleet sizes, which are compared and inferences drawn. The developed approach can serve as a template for decision support tool for call-taxi service providers.

Keywords: Call-Taxi, Fleet Sizing, Petri Net, Simulation Modeling.

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1. Introduction

Call-taxi, traditionally a telephone call based taxi hiring service, is a new type of urban transportation system that has seen tremendous growth during the last two decades for intra-city travel in many cities. Over the years, it has transformed to a smartphone app based service where a customer requiring a transportation service makes a request through a smartphone app. Round the clock service, timely service and reasonable are some of the drivers for the success of call taxi service for intra-city commuting (Devanathan, 2013). The decreased availability of parking spaces in growing cities also make call-taxi a more preferred mode for intra-city commuting.

Once a request comes, the call-taxi service provider need to assign a taxi to the customer, mostly based on its proximity to customer's pick-up location. The computerized system decides on the assignment of a taxi to the customer. The assigned call-taxi need not always be at the customer's pick-up location and accordingly it has to travel some distance to reach the customer. The traveling of empty call taxi to reach a customer is called deadheading, which is not very desirable but happens out of necessity. At times, a customer has to wait for long if the call-taxi assigned is located at a region

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away from customer's pick-up location. This waiting time directly reflects the level of customer service on how quick a call-taxi can be made available to a customer. Hence, having the right fleet size helps in striking a balance between deadheading and prompt taxi availability.

Fleet sizing decisions are not just limited to call taxis but span across all modes of transportation. Maisiuk and Gribkovskaia (2014) developed discrete event simulation model for offshore supply vessel fleet sizing with stochastic sailing times. Godwin et al. (2008) developed a Petri Net based simulation model for determining locomotive fleet size and its deadheading policy in a rail network. Tank car fleet sizing in the context of chemical supply chain was studied by Sha and Srinivasan (2016) where an agent-based simulation model is used for evaluating various fleet sizes and routing policies. Container fleet sizing and their repositioning in shipping systems was studied by Dong and Song (2009) where a simulation based optimization that uses Genetic Algorithm and Evolution Strategy is developed for determining container fleet size and its repositioning in a multi-vessel, multi-port and multi-voyage shipping systems. George and Xia (2011) developed an analytical model for optimal fleet sizing of a vehicle rental company where they viewed the problem as a steady state based optimization and used an iterative algorithm to find the solution.

Taxi system has evolved over a century with different markets having varying characteristics. Aarhaug and Skollerud (2014) studied the differences in regulations of taxis by market segments and various types of taxi services. Salanova et al. (2011) reviewed models developed for taxicab problems from the perspectives of regulation issues, operational organization and market organization. Salanova et al. (2014) observed that the variation of taxi fleet size and their passenger demand can influence the customer-search behavior of empty taxis. They reviewed the formulations used to estimate various metrics of taxi services in urban regions such as fleet size, generalized cost and optimum fleet.

Customer search behavior for taxis do vary by individuals and its variations does have an impact on the services planning. Wong et al. (2014) developed a logit model to predict such behavior. The prediction of the passenger service choice behavior between shuttle bus and taxi service for the last mile travel was modelled by Wang et al. (2013). The imbalance between supply and demand leads to lost service or deadheading of call taxis. Yang and Wong (1998) modelled the movement of vacant and occupied taxis in search of customers and found that taxi utilization level is correlated with the customer waiting time. The travel times in a city road network is always stochastic with unpredictable delays. In a congested road network, the shortest distance path need not be shortest time path (Godwin et al., 2016). The variation in travel time of a taxi in a city road network greatly influences the scheduling decisions. Scheduling dial-a-ride system in a city road network with time dependent and stochastic delays was modelled by Xiang et al. (2008). A metaheuristic-based solution approach for dynamic dial-a-ride problem in a road network with stochastic and time dependent travel speeds was developed by Schilde et al. (2014).

Given the stochastic nature of customer requests and travel times, simulation modelling has been one of the widely used approaches for taxi fleet sizing. Maciejewski and Bischoff (2015) simulated taxi services and developed two heuristics that dynamically dispatches taxis within the simulation. The effect of passenger travel demand on the quality of service rendered by taxis was analyzed using an agent-based simulation model by Kim et al. (2011). The study showed the importance of having symmetric travel demand pattern for having a higher service quality for taxi drivers and customers. An agent-based simulation that uses optimization models was developed by Lammoglia et al. (2012) to compare two types of taxi services and capture their cooperation and flexibility in responding to dynamic service requests. Grau and Romeu (2015) considered temporal and spatial dimensions of supply and demand of taxi services in urban area and conducted a simulation study.

This paper develops a Petri Net embedded simulation model of a generic call-taxi system and applies simulation via optimization approach to determine the fleet size. Three optimization models are developed and the resulting systems are compared. The developed approach can be customized and adopted for any intra-city call-taxi system for tactical planning of fleet size.

The rest of the paper is organized as follows. Section 2 describes the call taxi system under consideration and develops a Petri Net representation of it. The simulation modelling methodology including optimization are described in Section 3. Section 4 discusses the computational results and the conclusions are summarized in Section 5.

2. Call-Taxi System

2.1. Call-Taxi Operation

Call-taxi is a new type of urban transportation system that has seen tremendous growth during the last couple of decade in many cities. A customer requiring a transportation service makes a request through a smartphone app. The call-taxi service system then assigns a taxi to the customer, mostly based on its proximity to customer's pick-up location. However, the call-taxi need not always be at the customer's pick-up location at the time of booking and often it has to travel through the city to reach it. Sometimes a customer has to wait for long if the call-taxi assigned is located at a location away from customer's pick-up location.

A tactical level decision problem for call-taxi service providers is to decide the required fleet size of call-taxis for a city on a given day for the forecasted demand that seeks a balance between customer service and fleet utilization. Most of the call-taxi operators have a non-asset based system and are dependent on individual taxis with drivers who have enrolled with them. A taxi operating on a given day needs to have a minimum business to make it viable for the taxi owner as there is a revenue sharing with the call-taxi service provider. Hence, for a forecasted demand on a given day, there needs to be an ideal fleet size of call-taxi available that ensures higher vehicle utilization and a lower customer waiting time. The forecasted demand for call-taxi is a function of several factors such as day of week, festive day, long weekend etc. The stochastic nature of travel times in a city road network makes it an even more challenging decision problem.

2.2. Petri Net Model

A Petri net is a graphical tool used to model discrete event systems to help understand their working. Murata (1989) presented a detailed tutorial for modeling a system using Petri nets. A Petri net uses *tokens* on a directed bipartite multi-graph to simulate the activities of a system that helps to better understand sequences, concurrences and confluences of activities. There are two types of nodes: places and transitions, where places represent conditions while transitions represent events. We name those places connected to a transition by an outbound directed arc as input places while places connected by an inbound directed arc from a transition are named as output places. The definition of input and output places is purely relative for the purpose of differentiating two places connected to a transition since an input place of a transition could be an output place for a different transition and vice-versa. There could also be instances where an input place also serves as an output place for a transition. A transition can have more than one input and output places. The tokens are present inside places and a transition connected to an input place can fire only if sufficient number of tokens is present in it. Firing of transition indicates occurrence of an event and in that process the token(s) from the input place(s) of the transition is transferred to the output place(s).

A schematic functioning of a call-taxi system is modeled using Petri net, which provides the necessary conditions for a taxi to be assigned by a dispatcher to pick a customer as well as the conditions for the taxi to be released by the customer. In a real world scenario, customers and vacant taxis would be located at various places in the city and each customer's pick-up and drop-off location could be different. However, for the sake of modeling simplicity, a generic representation of the call-taxi system is modeled. The Petri net model for call-taxi system is shown in Figure 1, and Table 1 describes the places and transactions used in it.

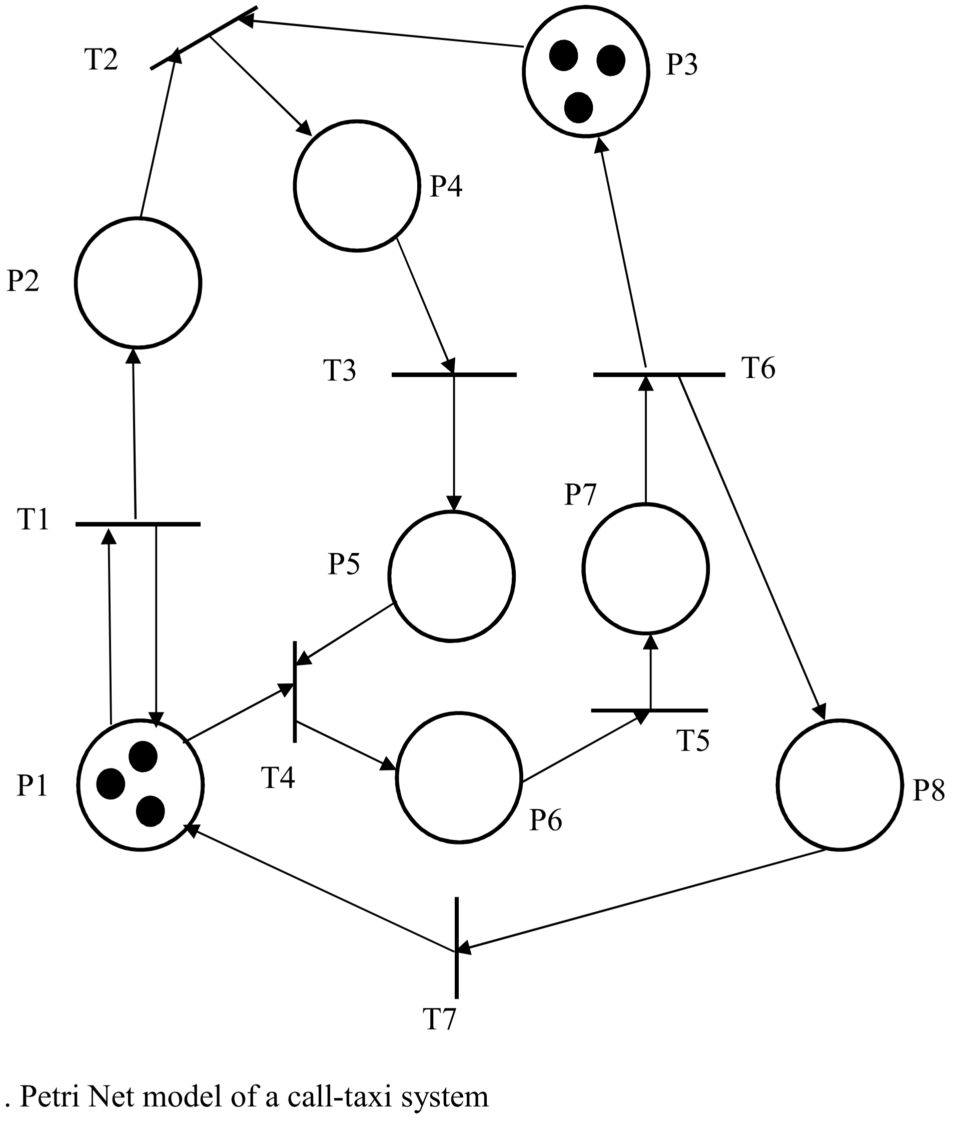

Figure 1. Petri Net model of a call-taxi system

Table 1. Description of places and transitions used in the call-taxi Petri Net model

Place/Transition	*Description*
P1	Customers in need of call-taxi
P2	Call-taxi request through the smartphone app
P3	Vacant taxis
P4	Taxi assigned to the customer
P5	Taxi at the pick-up location of the customer
P6	Taxi with the customer at the pick-up location
P7	Taxi at the drop-off location of the customer
P8	Disembarked customers at the drop-off location
T1	Customer making a request for call-taxi
T2	Dispatcher assigning a call taxi to the customer
T3	Taxi traveling to the customer's pick-up location
T4	Customer boarding the taxi
T5	Taxi with the customer traveling to the drop-off location
T6	Customer disembarking from the taxi at the drop-off location
T7	Customer deciding to book a call-taxi

The tokens in places P1 and P3 indicate the number of customers requiring taxi and the number of vacant taxis respectively. Transition T1 depicts the process of booking a taxi, which results in a token moved from P1 to P2. For transition T2 to fire, two conditions have to be satisfied: (a) there is a request from a customer for a taxi, which is indicated by the presence of a token in P2, and (b) a vacant taxi is available as indicated by the presence of a token in P3. Once a taxi is assigned to a customer (P4), it has to travel through the city road network (T3) to reach the customer's pick-up location. It should be noted that it does take some time for the taxi to reach a customer's pick-up point based on the location of taxi when it is assigned to the customer. The customer can board the taxi (T4) only after the taxi has reached the customer's pick-up location (P5). The taxi would travel (T5) to the drop-off location only after the passenger has boarded the taxi at the pick-up location (P6). The taxi is available for the next customer (P3) only after the current customer disembarks from it (T6). Customers in the drop-off location (P8) can book a taxi after a decision is made to do so (T7) while the empty taxi is now part of the number vacant taxis available for assignment (P3).

3. Methodology

3.1. Assumptions and System Characteristics

The following are the assumptions and system characteristics while modeling the call-taxi system.

a. There is only one type of call-taxi
b. A city road network is divided into zones.
c. The travel time for a call-taxi from one zone to another follow statistical distributions.
d. A customer's demand for call-taxi is defined with the zone of pick-up location and the zone of drop-off location.
e. Customer demand for call-taxi follow statistical distributions.
f. A call-taxi is permitted to deadhead between zones to pick-up a customer.

g. The deadheading within a zone to pick-up a customer incurs a minimum travel time, which is also statistically distributed.

For a given customer request, the nearest available call-taxi will be assigned.

3.2. Simulation Modelling

The Petri Net model described in section 2.2 is used as the basis for developing a discrete event simulation model. The simulation model is developed in such a way that the Petri Net model is applied to any pair of zones in a city road network. The modeling is done using Arena simulation software and a screenshot of it is shown in Figure 2.

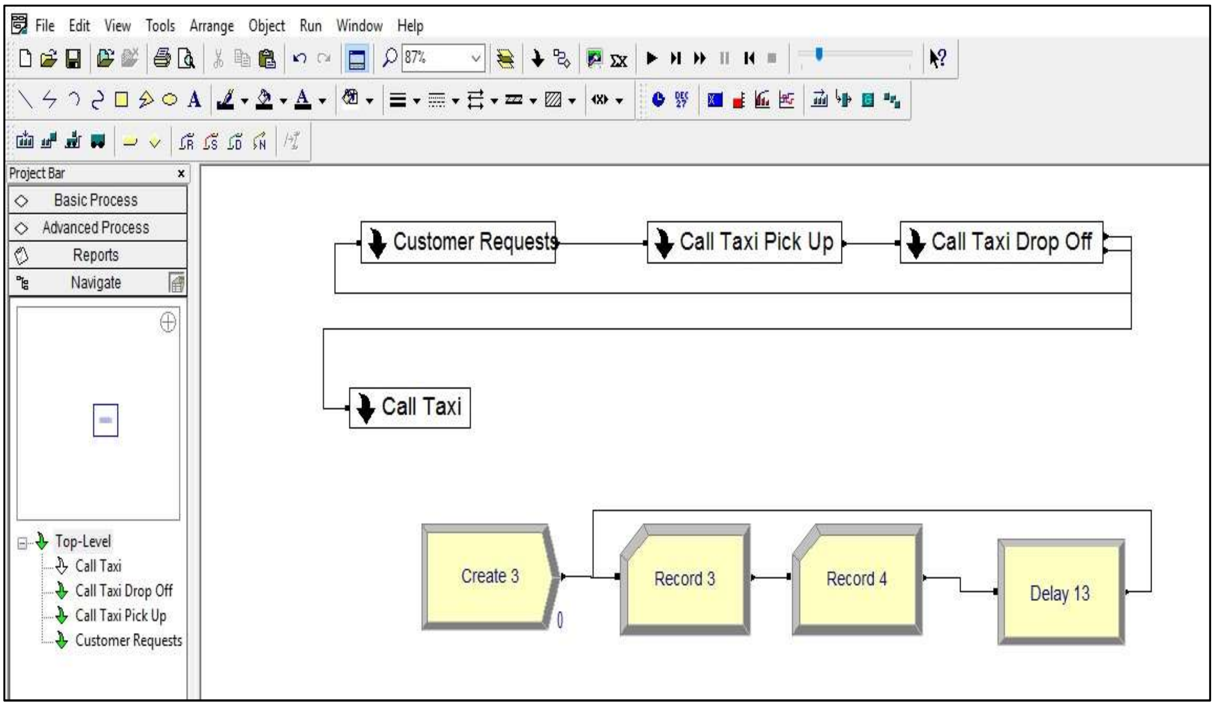

Figure 2. Simulation Model of Call-Taxi System using Arena Simulation Software

The simulation model requires fleet size, customer arrival distribution at each zone, and call-taxi travel time distribution between zones as inputs. The model records performance measures in terms of order pick-up rate, call-taxi running duration, call-taxi deadheading duration and customer waiting time. The output performance measures are detailed in the next sub-section.

3.3. Optimization via Simulation

Simulation is an evaluative tool that would evaluate the system for a given input by taking into consideration the stochastic nature of activities. In the call-taxi system, customer demand generation and the travel time of call-taxis between zones in a city road network are stochastic. These stochastic characteristics are difficult to model in normal optimization but there is a way to integrate simulation with optimization. In an optimization model, the decision variables are systematically perturbed and for each combination of them, the objective function is evaluated. The optimization process stops when there is no further improvement is observed in the objective function value. In a simulation model, the input parameters are the decision variables and the simulation run is the objective function evaluation that considers the stochastic nature of the system. This mapping of input parameters and simulation run to decision variables and objective function evaluation of optimization model respectively helps in implementing optimization via simulation. A schematic representation of optimization via simulation is shown in Figure 3.

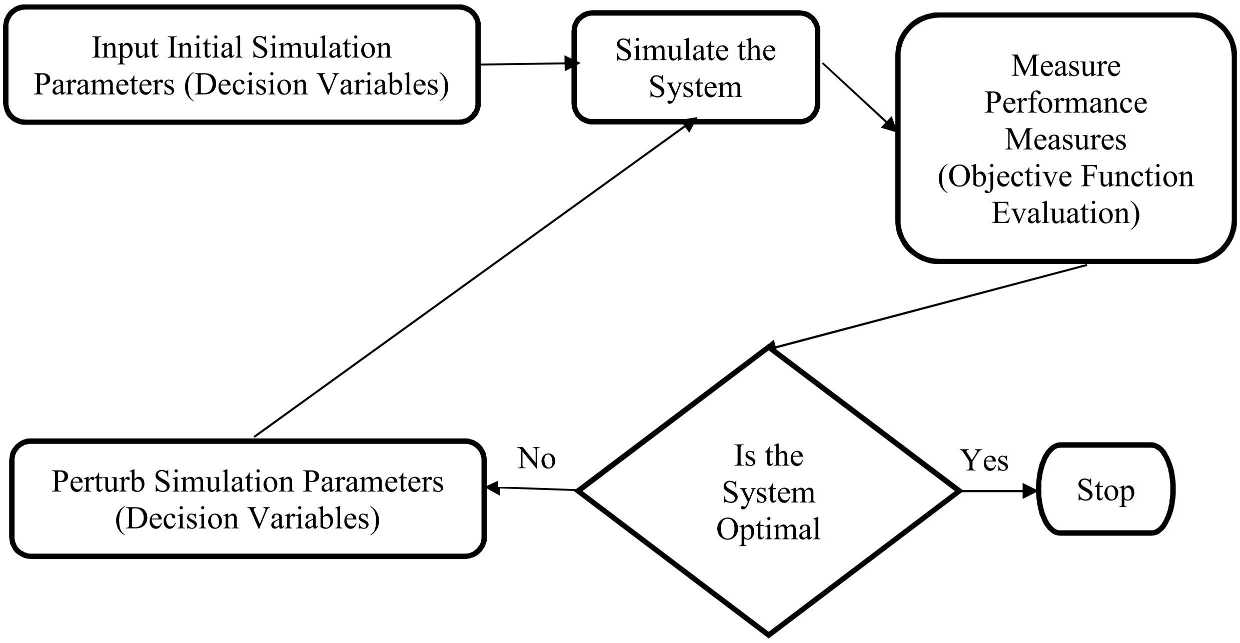

Figure 3. Optimization via Simulation

The process of optimization via simulation requires the desired objective functions to be defined along with constraints, if any, so that the system can optimize accordingly. In this study, three optimization models are defined for experimentation.

Notations:
N = fleet size of call-taxi
t = time duration that will be used as base for measuring performance (simulation run length)
WT = average waiting time of customers for call-taxi pick-up measured from the time of booking (averaged across call-taxis)
O_t = number of customers picked up in time duration t
RT_t = cumulative running time of call-taxis with customers in time duration t
DT_t = cumulative deadheading time of call-taxis without customers in time duration t
α = minimum number of customers that have to be picked up in time duration t
β = maximum permitted average waiting time of customers for pick-up from the time of booking
γ = maximum permitted ratio of DT_t and RT_t

Model 1

$$\text{Maximize } O_t \tag{1}$$

Model 1 is an unconstrained optimization model that tries to maximize the number of customers served and accordingly decides the fleet size. The objection function (1) maximizes the number of customers picked up during the course of simulation run and perturbs the fleet size to achieve it. However, the model does not consider the waiting time of customers and the proportion of deadheading by call taxis. This model strives to increase the overall market share and revenue for the call-taxi operator but could potentially dilute the individual call-taxi's revenue.

Model 2

$$\text{Minimize } WT \tag{2}$$

Model 2 is an unconstrained optimization model that tries to minimize the average waiting time of customers for call-taxis and accordingly decides the fleet size. However, the model does not consider the number of customers served and the proportion of deadheading by call taxis. This model strives to provide the best possible service to customers but may result in larger fleet size and excess deadheading.

Model 3

$$\text{Minimize } N \tag{3}$$

Subject to

$$O_t \geq \alpha \tag{4}$$

$$WT \leq \beta \tag{5}$$

$$\frac{DT_t}{RT_t} \leq \gamma \tag{6}$$

Model 3 is a constrained optimization models that aims to find the minimum possible fleet size to meet certain requirements on number of customers served, average waiting times of customers and the proportion of deadheading by call taxis.

The Arena Simulation software used for developing the simulation model has a feature called OptQuest for implementing optimization via simulation. The three optimization models can be embedded through OptQuest while running the simulation model. The next section provides the details of the computational study.

4. Computational Results and Discussion

Petri Net embedded simulation model of a call-taxi system is developed for a hypothetical city road network that is divided into five demand/supply zones (Figure 4). The demand for call-taxi in the system is exponentially distributed with a mean inter-arrival time of 5 minutes and the probability of this demand getting generated from the zones are 0.1, 0.15, 0.20, 0.25 and 0.30 for Zone 1, Zone 2, Zone 3, Zone 4 and Zone 5 respectively. Customer request pattern are probabilistic and travel time between any two regions in the city road network follow statistical distributions (Tables 2 and 3). There is a minimal travel time for a call-taxi to reach a customer's pick-up location even if it is present in the same zone.

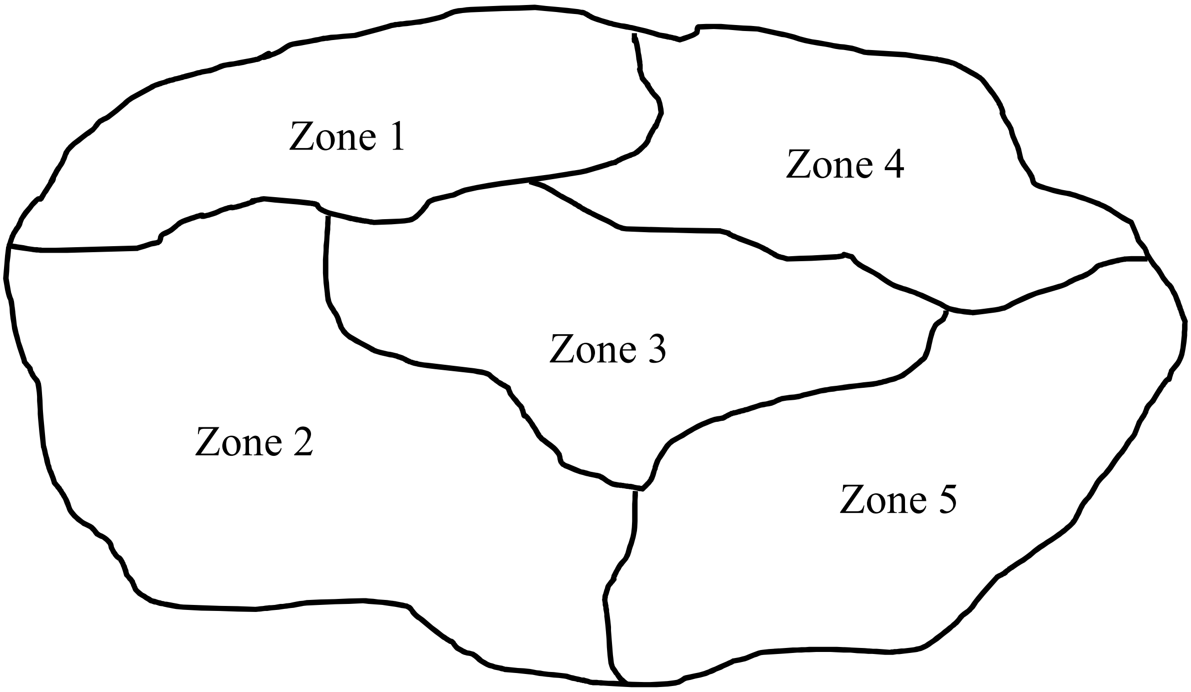

Figure 4. Representation of the City Road Network

Table 2. Probability of Drop-Off Zone from Pick-Up Zone for a Generated Demand

Pick-Up Zone	Drop-Off Zone				
	Zone 1	*Zone 2*	*Zone 3*	*Zone 4*	*Zone 5*
Zone 1	0.05	0.15	0.30	0.20	0.30
Zone 2	0.20	0.03	0.22	0.30	0.25
Zone 3	0.35	0.05	0.18	0.27	0.15
Zone 4	0.10	0.15	0.20	0.25	0.30
Zone 5	0.30	0.25	0.10	0.23	0.12

Table 3. Mean Value in Minutes for the Exponentially Distributed Travel Times

Pick-Up Zone	Drop-Off Zone				
	Zone 1	*Zone 2*	*Zone 3*	*Zone 4*	*Zone 5*
Zone 1	5	10	15	20	15
Zone 2	12	5	13	17	20
Zone 3	17	14	5	11	14
Zone 4	18	19	15	5	15
Zone 5	14	22	13	19	5

A call-taxi assigned to the generated demand at a zone (pick-up location) is routed to one of the zones (drop-off location) based on the probability in Table 2 and the travel time in Table 3. The call-taxi also deadheads to the pick-up location if it is not present at that location when the assignment happens. The outputs of the simulation for the three optimization models are summarized in Table 4.

Table 4. Output of the Optimization Models

Decision Variable and Performance Metrics	Optimization Model		
	Model 1	*Model 2*	*Model 3* ($\alpha = 180, \beta = 6, \gamma = 0.5$)
N	24	29	28
$\boldsymbol{O_t}$	189	185	186
$\boldsymbol{WT}$	6.051	5.89	5.97
$\boldsymbol{RT_t}$	1220.44	1248.82	1226.57
$\boldsymbol{DT_t}$	598.83	542.25	573.06

It is observed that there are tradeoffs between the models in terms of performance measures and the required fleet size. Model 3 is the more configurable version where the actual parameters could be customized based on the requirements for a day. For example, when demand is very high on a given day, customers would be willing to wait for some additional time than the normal and what really matters to them in this context is an assurance that they would be picked up. Model 3 also achieves a better balance between customer service and call-taxi utilization. We note that the computational study is just demonstrative on how optimization via simulation can be applied for call-taxi fleet sizing. The configurable nature of the Petri Net embedded simulation model makes it amicable for customization and scalable for real life implementation.

5. Conclusion

This paper developed a Petri Net representation of call-taxi system that was then embedded into a simulation model. Petri Net modeling helps in conceptualizing complex system with sequences, concurrences and confluences of activities such as the call-taxi system that can be further translated into simulation models. The concept of optimization via simulation is implemented for call-taxi fleet sizing by developing three optimization models. The simulation model and the optimization models are configurable, customizable and scalable for real world implementation. In addition, more optimization models can be defined based on the actual requirement of the system. The developed approach in the paper could serve as a template for coming up with a simulation based decision support system that has plenty of applications in real life where many things are stochastic in nature. Quality and accuracy customer demand pattern and the travel time estimates has a direct impact on the validity of the outputs of the optimization models. Hence, equal efforts must be put to ensure the quality and accuracy of input data while developing and implementing the simulation model in real world.

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