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Proposing a Train Speed Profile Generation Method in Railway Signalling Systems Based on Internet of Things (IoT): Performance and Stability Assurance

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Abstract

In the past years, the Internet of Things has created tremendous changes in various industries, including transportation systems and especially railway signalling systems. One of the issues that is very important in signalling systems is the design of trains speed profiles to control the movement of trains based on the defined headway and line conditions such as other trains positions. Information packet loss is one of the issues that should be considered in the field of Internet of Things and if this issue occurs, effective solutions should be adopted in the system for optimal performance. In this article, a method for generating speed profiles for moving block signalling systems in case of packet loss is proposed and the stability of the system is checked using the proposed method. Finally, we will implement the proposed method on a part of a railway line and examine the results.

Keywords: Internet of Things (IoT), Railway Signalling Systems, Trains Speed Profile, Stability Assurance, Information Packet Loss

1. Introduction

In recent years, the Internet of Things has been used in many industries. Railway signalling systems have undergone many changes with the advent of the Internet of Things (Walsh and Ye, 2001). So far, researchers have presented various distributed interlocking systems based on the Internet of Things to realize intelligent transportation systems. One of the problems in the design of these systems is the design of the train speed profile. Designing the speed profile for trains is done with various methods (Jong and Chang, 2005). One of these ways is design based on optimizing energy consumption (Cheng, Yin and Yang, 2021), (Zhong, Li, Xu and Zhang, 2020), (Aradi, Bécsi and Gáspár, 2013). The parameters that are effective in the design of the speed profile in

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signalling systems based on the Internet of Things and wireless sensor networks include the delay in sending information, noise, disturbance, bias of measurement devices, and loss of information packets. Therefore, the systems designed to provide the speed profile should have effective solutions for the mentioned problems. So far, methods have been provided in relation to delay and how to deal with it in wireless sensor networks (Peng and Sun, 2020), (Yue, Han and Lam, 2005), and solutions have also been provided for the loss of information packets (Xiong and Lam, 2007), (Liu, Wu, Yue and Park, 2019), (Wang, Song, Zhao, Sun and Zhuang, 2019), (Azimi-Sadjadi, 2003). Trains run on railway lines based on a time scheduling. Based on this schedule, the trains send their requests to the Interlocking system to start moving, and if the conditions are met in terms of safety, the trains are given movement authority (MA). In case of issuance of MA, according to the position of other trains, the speed profile is designed trains to move to the next station. From the time the train starts moving to stop, the speed profile will always be updated according to the information received from the interlocking system and other trains. The purpose of this process is to achieve the maximum use of the line capacity and the minimum headway.

In this article, we will examine how to design the speed profile to achieve the minimum headway and also discuss the stability of the system. Considering that the loss of information packets is one of the important issues in the field of Internet of Things, solutions should be adopted for the times when this happens. How to design the speed profile in case of data packet loss is another issue that we will discuss later. Checking the stability of the system in these conditions is also researched and the conditions of stability will be stated. To design the speed profile and write the related equations. Suppose the goal is to reach the lowest headway. In this case, according to headway definition, the time between the arrival of two consecutive trains to a certain station should be minimized. Therefore, we will have a minimum time optimization problem whose cost function is as (1).

$$J = t_f - t_0 = \int_{t_0}^{t_f} dt \tag{1}$$

The answer to this problem for the train movement is that first the train starts moving with the highest acceleration to reach the maximum allowed speed, and then brakes and stops with the highest possible acceleration. The issue here is that there is a speed limit for trains in each section of the line and it cannot be exceeded. This maximum speed is determined according to the conditions of the line, including the gradient of the line.

2. Generating Speed Profile

To realize the minimum time optimization problem, the movement of trains is done in three phases. In this strategy, the train starts moving with the highest acceleration and when the train speed reaches the maximum possible value, it continues to move with the same speed and finally stops with the highest acceleration (Locatelli and Sieniutycz, 2002). Considering the positive and negative accelerations of the train and the maximum speed, the parameters that are unknown in this problem are the times when changes in the train speed should be applied. The velocity equations related to this movement are as (2).

$$\begin{cases} v(t) = a_a(t - t_0), & t_0 \le t \le t_a \\ v(t) = v_m, & t_a < t \le t_b \\ v(t) = a_b(t - t_b) + v_m, & t_b < t \le t_f \end{cases} \tag{2}$$

t_0 : Time of starting the movement

t_a : Time of reaching the maximum speed

t_b : Time of starting to apply the brakes

t_f : Total duration of the train movement

a_a : Maximum positive acceleration

a_b : Maximum negative acceleration

v_m : Maximum allowed speed of the train

Also, the time to reach the maximum speed and the maximum speed are as (3).

$$t_a = \frac{v_m}{a_a} + t_0, v_m = a_a(t_a - t_0) \tag{3}$$

According to (3), following relations are written to obtain unknown parameters as (4).

$$\begin{cases} v(t_a) = v_m = a_a(t_a - t_0) \\ v(t_b) = v_m = a_a(t_a - t_0) \\ v(t_f) = a_b(t_f - t_b) + v_m \end{cases} \tag{4}$$

Considering $v(t_f) = 0$, for the start times of braking and the duration of the entire movement, (5) is valid.

$$a_b(t_f - t_b) + v_m = 0 \tag{5}$$

On the other hand, the equations related to the position of the train are written as (6).

$$\begin{cases} x(t) = 0.5a_a(t - t_0)^2 + x(t_0), & t_0 \le t \le t_a \\ x(t) = v_m(t - t_a) + x_a, & t_a < t \le t_b \\ x(t) = 0.5a_b(t - t_b)^2 + v_m(t - t_b) + x_b, & t_b < t \le t_f \end{cases} \tag{6}$$

x_a : Position of the train at the time of reaching the maximum allowed speed

x_b : Position of the train at the time of starting to brake

x_f : Final Position of the train

To calculate the unknowns of the equations, we must determine the movement strategy. There are two strategies to move the train from one station to the next. In the first case, suppose the train is at the station and intends to move to the next station. The movement strategy in this case is assumed that as long as there is a train along the route between two stations, the train is not allowed to move until the entire route is free. In this case, the length of the train can travel will be exactly equal to the distance between two stations. Therefore, (7) will be valid.

$$\begin{cases} x_a = 0.5a_a(t_a - t_0)^2 \\ x_b = v_m(t_b - t_a) + x_a = v_m(t_b - t_a) + 0.5a_a(t_a - t_0)^2 \\ x_f = 0.5a_b(t_f - t_b)^2 + v_m(t_f - t_b) + x_b \end{cases} \tag{7}$$

According to these equations and considering that the final position in this case is known and equal to the position of the next station, (8) is written.

$$x_b = x_f - 0.5a_b(t_f - t_b)^2 - v_m(t_f - t_b) \tag{8}$$

By combining (7) and (8), we get (9).

$$0.5a_b(t_f - t_b)^2 + v_m t_f = x_f - 0.5a_a t_a^{\ 2} + v_m t_a \tag{9}$$

As a result, they are obtained by solving (10) and the speed profile is designed.

$$\begin{cases} a_b(t_f - t_b) + v_m = 0 \\ 0.5a_b(t_f - t_b)^2 + v_m t_f = x_f - 0.5a_a t_a^{\ 2} + v_m t_a \end{cases} \tag{10}$$

In the second case, the movement strategy is such that each of the trains, regardless of the position of the front train, can move up to a safe distance from it. The speed equations related to this movement are as (11).

$$\begin{cases} v_i(t) = a_{i,a}(t - t_0), & t_0 \le t \le t_a \\ v_i(t) = v_m, & t_a < t \le t_b \ , \ t_a = \dfrac{v_m}{a_a} + t_0,\ v_m = a_{i,a}(t_a - t_0) \\ v_i(t) = a_{i,b}(t - t_b) + v_m, & t_b < t \le t_f \end{cases} \tag{11}$$

$a_{i,a}$: Maximum positive acceleration of the i-th train

$a_{i,b}$: Maximum negative acceleration of the i-th train

According to (11), (12) is written to obtain unknown parameters.

$$\begin{cases} v_i(t_a) = v_m = a_{i,a}(t_a - t_0) \\ v_i(t_b) = v_m = a_{i,a}(t_a - t_0) \\ v_i(t_f) = a_{i,b}(t_f - t_b) + v_m = 0 \end{cases} \tag{12}$$

The (13) equation is established for the relation between the starting time of braking and the duration of the entire movement.

$$a_{i,b}(t_f - t_b) + v_m = 0 \tag{13}$$

In this case, the equations related to the position of the trains will be as (14).

$$\begin{cases} x_i(t) = 0.5a_{i,a}(t - t_0)^2 + x_i(t_0), & t_0 \le t \le t_a \\ x_i(t) = v_m(t - t_a) + x_{i,a}, & t_a < t \le t_b \\ x_i(t) = 0.5a_{i,b}(t - t_b)^2 + v_m(t - t_b) + x_{i,b}, & t_b < t \le t_f \end{cases} \tag{14}$$

x_i : Real-time position

$x_{i,a}$: Position when the maximum speed is reached

$x_{i,b}$: Position when the i-th train starts braking

According to (14), (15) is written for i-th train position when it starts braking.

$$x_{i,b} = x_{i+1}(t) - x_d - 0.5a_{i,b}(t_f - t_b)^2 - v_m(t_f - t_b),\ \ x_{i,f} = x_{i+1}(t) - x_d \tag{15}$$

x_d : Safe distance

$x_{i,f}$: Final position of i-th train

By combining (14) and (15), we get (16).

$$0.5a_{i,b}(t_f - t_b)^2 + v_m t_f = x_{i+1}(t) - x_d - 0.5a_{i,a}t_a^{\ 2} + v_m t_a \tag{16}$$

Using the time-independent equation for the entire path between two stations, (17) is obtained.

$$\frac{v_m^{\ 2}}{2a_{i,a}} + v_m(t_b - t_a) - \frac{v_m^{\ 2}}{2a_{i,b}} = x_{i+1}(t) - x_d \tag{17}$$

As a result, by receiving the position of the front train and solving (18), the speed profile is designed.

$$\begin{cases} v_m = a_{i,a}(t_a - t_0) \\ a_{i,b}(t_f - t_b) + v_m = 0 \\ 0.5a_{i,b}(t_f - t_b)^2 + v_m t_f = x_{i+1}(t) - x_d - 0.5a_{i,a}t_a^{\ 2} + v_m t_a \\ \dfrac{v_m^{\ 2}}{2a_{i,a}} + v_m(t_b - t_a) - \dfrac{v_m^{\ 2}}{2a_{i,b}} = x_{i+1}(t) - x_d \end{cases} \tag{18}$$

Suppose the state space of the system is as (19).

$$X_i(k+1) = AX_i(k) + Bu_i(k),\ \ X_i(k) = \begin{bmatrix} x_i(k) \\ v_i(k) \end{bmatrix} \tag{19}$$

$X_i(k)$: Position and speed of i-th train

$u_i(k)$: Input of i-th train's position and speed equations

Finally, according to the definition of the three phases of acceleration, constant speed and braking, the input of the system will be as (20).

$$u_i(k) = \begin{cases} a_{i,a}, & t_0 \le t \le t_a \\ 0, & t_a < t \le t_b \\ a_{i,b}, & t_b < t \le t_f \end{cases} \tag{20}$$

3. Information Packet Loss

In this section, we assume that the information sent by the sensors during transmission through the IoT network may be lost with a probability factor that is called the loss of information packets. The sampler discretizes the system information and sends it to the controller through the IoT network. The zero-order hold block (ZOH), which holds its input for the specified sample period, also receives controller information through the IoT network (Ge, Yang and Han, 2017).

Set $\varphi = \{i_1, i_2, \ldots\}$ is a set of time series, where i_k refers to the number of successful periods from the sampler to ZOH. Therefore, $i_{k+1} - i_k$ means the distance between two successful time periods, and we define this maximum distance as (21).

$$s \triangleq \max(i_{k+1} - i_k) \tag{21}$$

We define the process of losing information packets as (22).

$$\begin{aligned} &\eta(i_k) \triangleq i_{k+1} - i_k : i_k \in \varphi \\ &\varphi \triangleq \{1, 2, \ldots, s\} \end{aligned} \tag{22}$$

Therefore, from ZOH's point of view, when data is lost, the healthy value received from the previous successful period must be preserved and this value continues until another healthy data is received in a subsequent period.

Now suppose that data is not received from a certain time, and in other words, information packets are lost. In this case, according to Figure 1, as soon as the information packets are lost and from the real time position of the train at the time of not receiving the information, until the time of correctly receiving the information again, the speed of the train is set to the value of the speed of the last received correct data. In other words, we consider the input of the system equal to zero from the time that data is not received. Now we check the equations in both cases defined in the previous section.

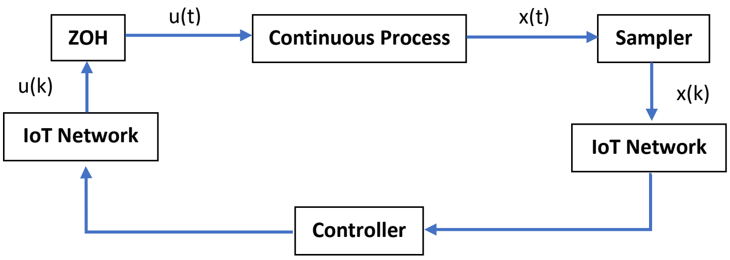

Figure 1: System Modelling in Case of Information Packet Loss

In the first case, where the movement strategy is such that the train can start moving when the path to the next station is free, the equations related to the speed profile and the position of the train should be such that it stops the train at the next station. Therefore, the speed equations will be as (23).

$$\begin{cases} v(t) = v_{pl}, & t_{pl} < t \le t_b \\ v(t) = a_b(t - t_b) + v_{pl}, & t_b < t \le t_f \end{cases} \tag{23}$$

t_b : Time that train starts to brake

a_b : Train brake acceleration

t_{pl} : Time that data is not received

v_{pl} : Last received correct data of train speed

Considering that the train stops at time t_f , as a result, relation $v(t_f) = 0\,m/s$ holds. The (24) is valid for the start times of braking and the duration of the entire movement.

$$a_b(t_f - t_b) + v_{pl} = 0 \tag{24}$$

Also, the equations related to the position of the train are as (25).

$$\begin{cases} x(t) = v_{pl}(t - t_{pl}) + x_{pl}, & t_{pl} < t \le t_b \\ x(t) = 0.5a_b(t - t_b)^2 + v_{pl}(t - t_b) + x_b, & t_b < t \le t_f \end{cases} \tag{25}$$

x_{pl} : Last received correct data of train position

Considering that the final destination of the train is the next station and its location is known, therefore (26) is written.

$$\begin{cases} x_b = v_{pl}(t_b - t_{pl}) + x_{pl} \\ x_f = 0.5a_b(t_f - t_b)^2 + v_{pl}(t_f - t_b) + x_b \end{cases} \tag{26}$$

x_b : Train position when it starts to brake

According to these equations and considering that the final position in this case is known and equal to the position of the next station, (27) is written for x_b .

$$x_b = x_f - 0.5a_b(t_f - t_b)^2 - v_{pl}(t_f - t_b) \tag{27}$$

By combining (26) and (27), we get (28).

$$0.5a_b(t_f - t_b)^2 + v_{pl}t_f = x_f - 0.5a_a t_a{}^2 + v_{pl}t_a \tag{28}$$

As a result, the speed profile is obtained according to relations (24) and (28).

In the second case, the movement strategy is such that each of the trains, regardless of the position of the front train, can move up to safe distance from it.

If the loss of information packets occurs in this state since the last received speed of the train remains constant until receiving the information again, the equations of this state are the same as the equations of the first state, but the final position of the train where it should stop is the safe distance away from the front train.

Therefore, by placing $x_{i,f} = x_{i+1}(t) - x_d$, all the equations of the first state will be valid here as well.

Suppose the state space of the system is as (29).

$$X_i(k+1) = AX_i(k) + Bu_i(k) \tag{29}$$

As a result, the input of the system from the time of information packet loss to the arrival of the destination will be as (30).

$$u_i(k) = \begin{cases} 0, & t_{pl} < t \le t_b \\ a_{i,b}, & t_b < t \le t_f \end{cases} \tag{30}$$

4. Stability Assurance

Suppose the system model is discrete in time in the following state space form as (31).

$$x(k+1) = Ax(k) + Bu(k) \tag{31}$$

Assume that the controller in the internet of things network is state feedback as (32).

$$u(k) = Kx(k), \tag{32}$$

By substituting (32) in (31), (33) is obtained.

$$x(k+1) = (A+BK)x(k) = A_d x(k) \tag{33}$$

Now, to check the stability of the system, we should propose a Lyapunov function. Consider the relation (34) to be the proposed Lyapunov function (Zhou, Hu, Zhu and Ma, 2021).

$$V(k) = x^T(k)Px(k) > 0 \tag{34}$$

To check the stability of the system, we form the time derivative of proposed Lyapunov function that must be negative for the stability of the system. The time derivative of Lyapunov function is written as (35).

$$V(k+1) - V(k) = x^T(k+1)Px(k+1) - x^T(k)Px(k) = x^T(k)[A_d^T PA_d - P]x(k) \tag{35}$$

According to the definition of Lyapunov stability, the system is asymptotically stable if and only if (36) holds.

$$(A+BK)^T P(A+BK) - P < 0, \;\; P > 0 \tag{36}$$

Where the values of P and K are unknown and relative to the values of them, which are unknown, it is non-linear because there is PBK in that term and it forms a bilinear inequality which is represented by BMI. As we know these relations are not convex and we should try to convert it into an LMI.

Remark: Schur Complement Lemma: Suppose that matrices A and C are symmetric and matrix A is positive definite and (37) holds.

$$U = C + B^T A^{-1} B < 0 \tag{37}$$

Then (38) will always be true.

$$U_1 = \begin{bmatrix} -A & B \\ B^T & C \end{bmatrix} < 0, \; U_2 = \begin{bmatrix} C & B^T \\ B & -A \end{bmatrix} < 0 \tag{38}$$

As a result, matrix U is called Schur Complement of A by U. According to (36) and the corresponding placement of system matrices with (38), (39) is obtained.

$$\begin{bmatrix} -P & A_d^T + K^T B_d^T \\ A_d + B_d K & -P^{-1} \end{bmatrix} < 0, \;\; P > 0 \tag{39}$$

Multiply both sides of the relation (39) from the left and right by $diag(P^{-1}, I)$ and consider the matrix Q as $Q = P^{-1}$. By applying these changes, (40) is obtained.

$$\begin{bmatrix} -Q & QA_d^T + QK^T B_d^T \\ A_d Q + B_d KQ & -Q \end{bmatrix} < 0, \; Q > 0 \tag{40}$$

Now, with definition $KQ = N$, (40) can be rewritten as (41).

$$\begin{bmatrix} -Q & QA_d^T + N^T B_d^T \\ A_d Q + B_d N & -Q \end{bmatrix} < 0,\ Q > 0 \tag{41}$$

where Q and N are unknown parameters and the relationship is linear with respect to them, and as a result, the (41) is an LMI.

In the following, in case of information packet loss, we will examine the stability of the system by considering the Lyapunov function proposed in (34).

A dynamical system is Lyapunov stable if all its responses, placing the initial state near the equilibrium point, remain forever around the equilibrium point.

Otherwise, the dynamic system is unstable. The system is asymptotically stable if all responses that start near the equilibrium point not only remain near it, but also converge to the equilibrium point as time tends to infinity. Asymptotic stability is a stronger form of stability.

The Lyapunov stability theorems provide sufficient conditions for the stability of the dynamic system, but these theorems do not state whether these stability conditions are necessary or not.

To check the stability of the designed system, we first estimate the Lyapunov functions and then we will check the stability using this method.

Now, if we call l, the time between two successful periods, the discrete state space of the system is stable if (42) holds.

$$\|x_0\| < \delta \ \rightarrow \ \|x(l, x_0)\| < \varepsilon \ \rightarrow \ \lim_{l \to \infty} \|x(l, x_0)\|^2 = 0 \tag{42}$$

To check the stability, we first define the Lyapunov function as (43).

$$\begin{cases} V(i_k) = x^T(i_k) P_{i_k - i_{k-1}} x(i_k) = x^T(i_k) P_i x(i_k) \\ V(l) \triangleq x^T(l) P_{l - i_k} x(l) \end{cases} \tag{43}$$

To calculate the time derivative of the proposed Lyapunov function, (44) can be written.

$$\begin{cases} V(i_{k+1}) = x^T(i_{k+1}) P_{i_{k+1} - i_k} x(i_{k+1}) = x^T(i_k)(A^j + B_j k)^T P_j (A^j + B_j k) P_i x(i_k) \\ V(i_{k+1}) - V(i_k) = x^T(i_k)[(A^j + B_j k)^T P_j (A^j + B_j k) - P_i] x(i_k) \end{cases} \tag{44}$$

A discrete-time state space system is asymptotically stable if there exists a matrix P_i such that (45) holds.

$$(A^j + B_j k)^T P_j (A^j + B_j k) - P_i < 0,\ B_j \triangleq \sum_{r=0}^{j-1} A^r B \ \rightarrow \ \lim_{i_k \to \infty} V(i_k) = 0 \tag{45}$$

Now suppose that relation $i_k + 1 \leq l \leq i_{k+1}$ holds true. Then (46) are obtained.

$$x(l) = (A^h + B_h k) x(i_k),\ B_h = \sum_{r=0}^{h-1} A^r B,\ h = l - i_k \tag{46}$$

As a result, (47) will hold for the Lyapunov function.

$$\begin{aligned} & V(l) - V(i_k) = x^T(i_k)[(A^h + B_h k)^T P_h (A^h + B_h k) - P_i] x(i_k) < 0 \\ & \rightarrow \ \lim_{l \to \infty} V(l) = 0 \end{aligned} \tag{47}$$

According to (47) and (38), (48) is obtained.

$$\begin{bmatrix} -P_i & (A^h)^T + K^T B_h^T \\ A^h + B_h K & -P_h^{-1} \end{bmatrix} < 0,\ P_i > 0,\ P_h > 0 \tag{48}$$

Considering the matrix Q as $Q = P_h^{-1}$, (49) is obtained, which is a LMI.

$$\begin{bmatrix} -P_i & (A^h)^T + K^T B_h^T \\ A^h + B_h K & -Q \end{bmatrix} < 0,\ P_i > 0,\ Q > 0 \tag{49}$$

5. Results

Suppose we want the profile of the speed and position for the movement of trains between two stations, the distance between which is 1469 meters, and the maximum speed allowed for the movement of trains on the route is 20 meters per second. Also suppose that the maximum positive acceleration of the train is 0.5 m/s^2 and the maximum negative acceleration of the train is -0.5 m/s^2. Considering that the goal is to obtain the profile of the speed and position of the train and considering the equations of motion of the trains, the state space matrices of the system are as (50).

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{50}$$

As explained earlier, two strategies can be adopted to determine how the trains move. In the first strategy, we assume that the train can move to the next station if the entire path to the destination is free. By solving the relevant equations, the times related to the three phases of acceleration, moving at a constant speed and starting to brake, as well as the input of the system are obtained as (51).

$$u(t) = \begin{cases} 0.5, & t_0 \le t \le t_a \\ 0, & t_a < t \le t_b \\ -0.5, & t_b < t \le t_f \end{cases},\ t_a = 28s,\ t_b = 74s,\ t_f = 104s \tag{51}$$

Therefore, in this case the speed and position profiles are designed as Figure 2.

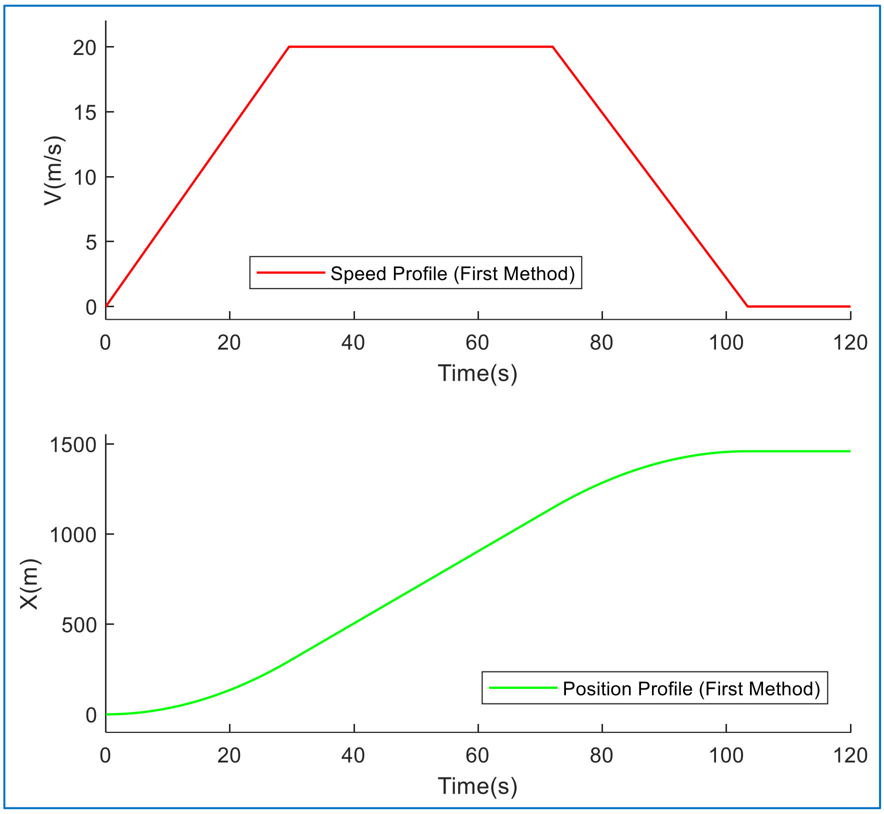

Figure 2: Speed and Position Profiles (1st Strategy)

In the second strategy, the train regardless of whether there is a train on the track ahead, enters the track after receiving permission to move and moves up to a safe distance from the front train, which is assumed to be $x_d = 100\,m$. Suppose that the front train has stopped at the next station platform and MA has not been issued. The train can move up to 100 meters before the front train and stop at $x_{i,f} = x_{i+1}(t) - x_d = 1369\,m$. The unknowns of the equations are calculated as (52).

$$u(t) = \begin{cases} 0.5, & t_0 \le t \le t_a \\ 0, & t_a < t \le t_b \; , \; t_a = 28s, \; t_b = 65s, \; t_f = 98s \\ -0.5, & t_b < t \le t_f \end{cases} \tag{52}$$

Therefore, in this case the speed and position profiles are designed as Figure 3.

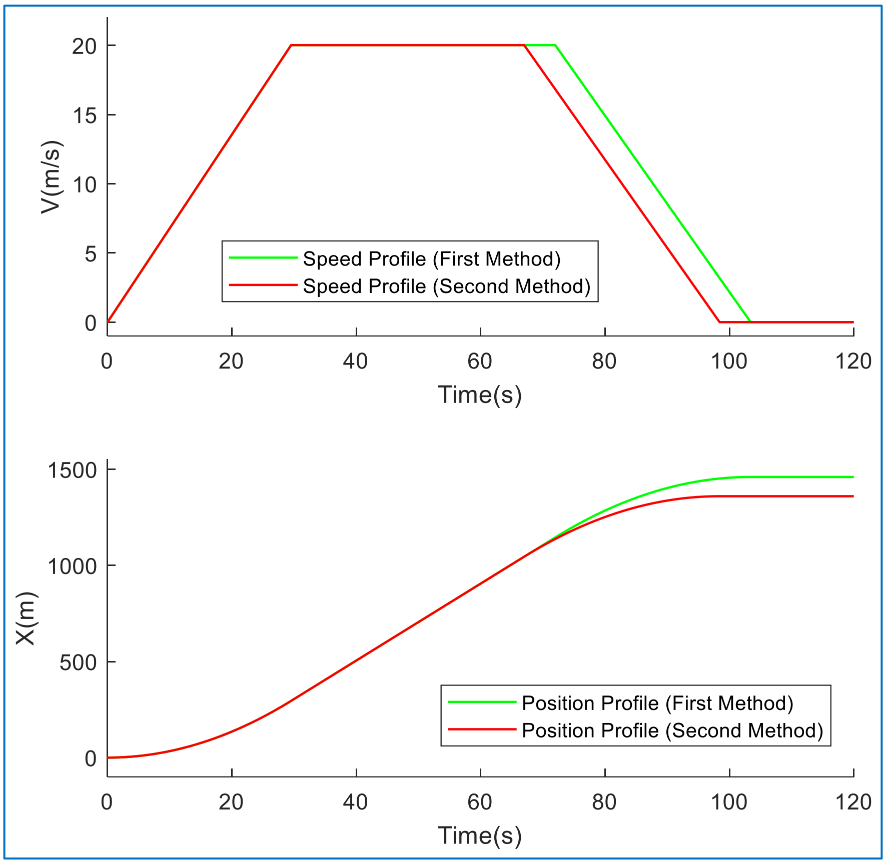

Figure 3: Speed and Position Profiles (1st and 2nd Strategies)

To check the stability, by solving (41), the unknown parameters are obtained as (53).

$$A_d = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \; B_d = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; K = -\begin{bmatrix} 0.8158 \\ 1.8217 \end{bmatrix}^T, \; P_i = \begin{bmatrix} 11.6773 & 10.5948 \\ 10.5948 & 12.2863 \end{bmatrix} \tag{53}$$

Suppose we have chosen the first strategy for design and after some time has passed since the start of the train movement, the data is not received correctly. In this case, according to the system modeling according to Figure 1, we consider the last received data until the arrival of the next data and set the speed of the train equal to the last received speed. In other words, as soon as the information packets are lost, we set the system input to zero, or in other words, no acceleration is applied to the train until the next information arrives. Next, to stop the train to the next station, we calculate the new speed and position profile according to the strategy described in this article and apply it to the train. By solving the related equations, the unknowns of the equations are obtained as (54).

$$u_i(k) = \begin{cases} 0, & t_{pl} < t \le t_b \\ -0.5, & t_b < t \le t_f \end{cases}$$
$$t_{pl} = 21s, \; t_b = 105s, \; t_f = 128s, \; v_{pl} = 14\ m/s, \; x_{pl} = 150m, \; x_f = 1469m \tag{54}$$

Therefore, in this case the speed and position profiles are designed as Figure 4.

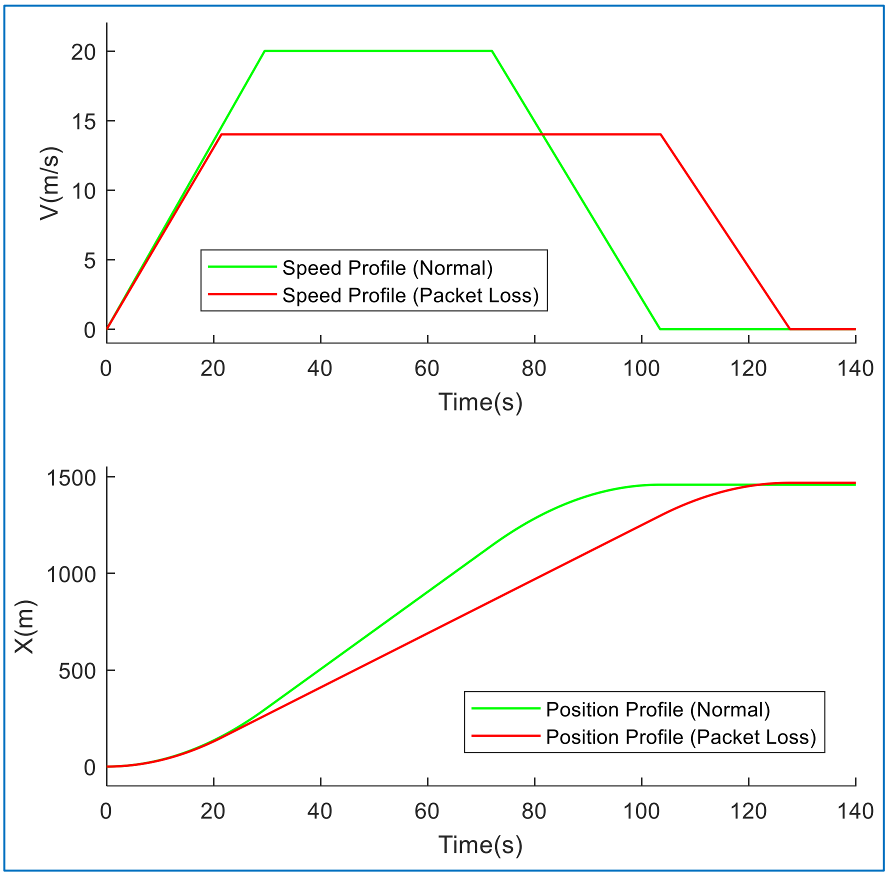

Figure 4: Speed and Position Profiles (1st Strategy: Normal and Packet Loss)

Consider the second strategy for designing the train's speed and position profile. In this case, the train can move up to 100 meters away from the front train. Suppose that the front train has stopped at the platform of the next station and the information about the speed and position of the train is lost as in the previous case and at the same time. Considering that the strategy is to keep the speed constant until the arrival of the next information, the equations of this problem are the same as the previous case, with the only difference that braking should be done earlier when the train stops at a distance of 100 meters from the front train. Finally, the unknowns of the system of equations are calculated as (55).

$$u_i(k)=\begin{cases}0, & t_{pl}<t\le t_b\\ -0.5, & t_b<t\le t_f\end{cases} \tag{55}$$

$$t_{pl}=21s,\ t_b=98s,\ t_f=121s,\ v_{pl}=14\ m/s,\ x_{pl}=150m,\ x_f=1369m$$

Therefore, in this case the speed and position profiles are designed as Figure 5.

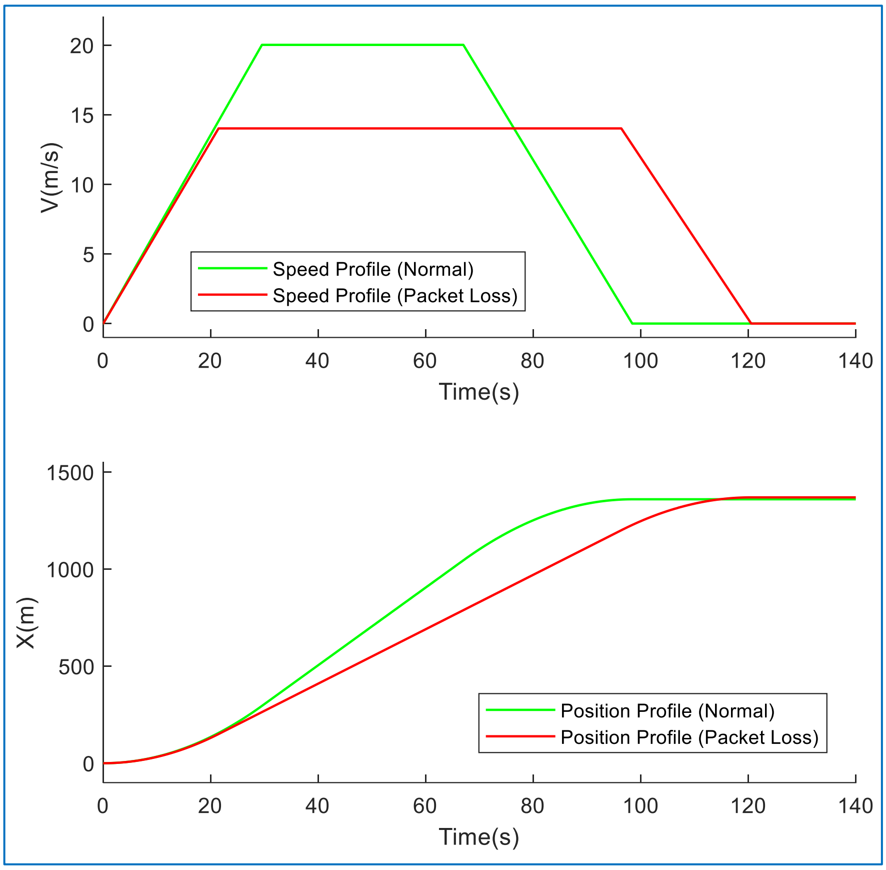

Figure 5: Speed and Position Profiles (2nd Strategy: Normal and Packet Loss)

In case of information packet loss, to check the stability, by solving (49), the unknowns of the equations are obtained as (56).

$$\begin{cases} t_{pl} = 21s \\ t_f = 121s \end{cases} \rightarrow h = 100,\ A^h = \begin{bmatrix} 1 & 100 \\ 0 & 1 \end{bmatrix},\ B_h = \begin{bmatrix} 4950 \\ 100 \end{bmatrix}$$
$$K = -\begin{bmatrix} 0.0212 \\ 0.2165 \end{bmatrix}^T,\ P_i = \begin{bmatrix} 11.6773 & 10.5948 \\ 10.5948 & 12.2863 \end{bmatrix},\ P_h = \begin{bmatrix} 0.0477 & 0 \\ 0 & 0.0477 \end{bmatrix} \quad (56)$$

In order to see the differences and compare the different investigated modes, the speed profile and position profile designed for the trains in all four modes consisting of the first and second strategies and considering the normal modes and the loss of information packets are drawn in Figure 6.

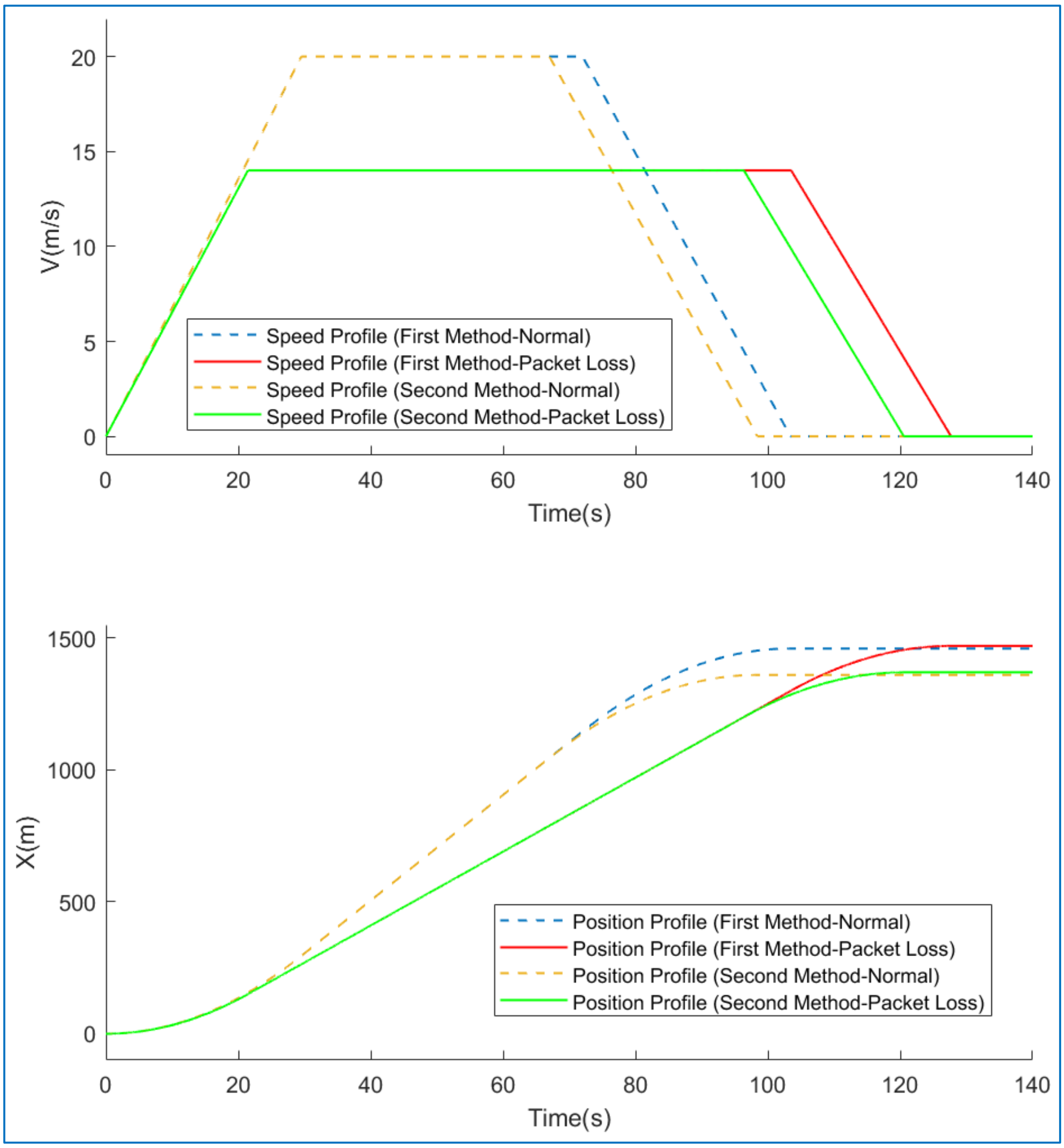

Figure 6: Speed and Position Profiles (1st and 2nd Strategies: Normal and Packet Loss)

6. Conclusions

In this article, a speed profile generation method for train movement was presented for use in signaling systems and automatic train protection systems in railway lines.

Two movement strategies were determined for the trains. The first strategy was defined in such a way that a train can move from one station to the next station only if there is no other train along the route. In the second strategy, each of the trains, regardless of the position of the front train, can move up to a safe distance from the front train. For each strategy, the speed profile was designed with the aim of minimizing the movement time. Next, the information packet loss problem was considered in the process of transferring data, and an optimal speed profile was designed for each strategy.

The stability of the designed system was analyzed using the Lyapunov stability method. For this purpose, Lyapunov functions were proposed for the normal mode as well as information packet loss, and the stability conditions in both modes were investigated according to the movement strategies. Finally, two stations of a railway line were considered and for each of the described modes, the optimal speed profile was designed and the stability of the system was guaranteed in each of them.

Issues that can be considered by researchers in this field in the future are the analysis of system stability in the presence of delay, bias of measuring devices, noise, and disturbance. Other parameters such as friction, environmental factors, etc. can also be included in the system model. Security and safety are other issues that should be investigated to ensure the correct operation of the designed system.

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