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Role of Multiple Leaders and their Dynamic Variables on Vehicular Following Behaviour in Mixed Traffic Condition

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Abstract

In mixed traffic condition, vehicles with varying static and dynamic characteristics share the same road space. The smaller vehicles frequently try to manoeuvre through the gaps available between larger vehicles. In this scenario, the use of conventional vehicle-following models which considers the effect of single (overlapping) leader may become inappropriate. The influence of multiple leaders and their combinations on the response of the subject vehicle is a crucial research gap identified from the literature survey. In the multiple leader cases, the follower does not merely adjust to the relative speed and spacing with the primary (overlapping) leader, but judiciously responds to subsidiary (non-overlapping) leader attributes also. Present study formulates a model structure for vehicle-following manoeuvre in mixed traffic by incorporating the multiple leader dynamic variables. The spatial orientation of multiple leaders along with vehicle types and dynamic variables are found to have a substantial role, thus resulting in realistic representation of mixed traffic. These characteristics have the potential to enhance existing following behaviour models and improve the realism of microscopic modelling in mixed traffic conditions. Simulation models that integrate these attributes could find practical applications in more accurate assessments of traffic management and operational strategies.

Keywords: vehicle-following models, mixed traffic condition, modified response-stimulus model, multiple leader dynamic variables.

1. Introduction

In mixed traffic condition, there can be parallel movement of vehicles in the same lane due to the wide variety of vehicles present on the road moving under weak lane disciplined condition. This can lead to a condition of multiple leaders being followed by the same subject vehicle. The conventional vehicle-following models (Gazis et al., 1959;

Reuschel, 1950; Pipes, 1953; Kometani and Sasaki, 1958; Forbes, 1963) developed for homogenous lane-based condition become inappropriate for such a scenario. The predominance of car-following in homogeneous traffic flow models is not appropriate for mixed traffic conditions, primarily due to the presence of several classes of vehicles besides car with varying static and dynamic characteristics. Parallel movement of vehicles leads to difficulty in defining the leading and following vehicles. Leading and following vehicles may run staggered without lane segregation. Drivers in heterogeneous non-lane-based traffic may not find lane discipline as an effective and advantageous means of utilising the road space. It is common to find smaller vehicles utilizing the lateral gap between the leading vehicles to move forward. The influence of multiple leaders on the longitudinal response of subject vehicles under strict following condition is yet to be investigated. Hence, there is need for framing a generalised vehicle-following behaviour to reflect the effect of different leading vehicle types and its dynamic characteristics on following behaviour. Mixed traffic conditions, in which various modes of transportation share the same road space, are also relevant to European countries. In European cities, mixed traffic conditions prevail as pedestrians, cyclists, cars, buses, and trams share the same road space. This allows for efficient use of limited urban space. The current paper discusses the scenario involving multiple leaders and how the dynamic variables between leaders and the subject vehicle influence the driving behaviour of the subject vehicle. This is relevant to traffic streams where the condition of multiple leaders exists, thus strengthening its connection to potential applications in European contexts.

A stimulus-response model is formulated considering the acceleration or deceleration of the subject vehicle as the response variable. The relative speed and spacing between the primary leader and subject vehicle are commonly considered as the stimuli. But this study establishes a new relationship between response and stimuli which is assumed to be affected by sensitivity parameters which are expected to vary based on multiple leader dynamic variables. Incorporating these terms into the longitudinal response equation creates a generalised model framework for multiple leaders. Therefore, the research objectives pursued in this study are as follows: (i) to develop a modified response-stimulus model structure incorporating multiple leader dynamic attributes (ii) to investigate the effect of sensitivity and stimulus of subject vehicle to primary leader across multiple leader spatial orientations (iii) to differentiate the driving behaviour of car and two-wheeler under each multiple leader categories. Incorporating the aforementioned mixed traffic attributes into the stimulus-response model can enhance the realism and improve the overall validity of the models. These enhanced models can be utilized for simulating non-lane-based traffic conditions characterized by heterogeneity, providing a more accurate representation of real-world scenarios. The findings of this study offer potential applications across various domains of traffic engineering, management, and operations.

2. Literature Review

The state-of-art response-stimulus model was formulated based on the concept that the response (acceleration) of a vehicle is a function of the stimulus it receives from the environment (Gazis et al., 1959). Microscopic modelling operates at the disaggregate level, focusing on the behaviour of individual vehicles and their interactions (Toledo et al., 2007). Acceleration models are fundamental in microscopic traffic simulation, and are used to predict the acceleration or deceleration of the following vehicle at various time instances (Reuschel, 1950). Several theories have been developed to model car-following behaviour, classified into five categories based on behavioural assumptions: collision avoidance models, stimulus-response models, psycho-physical models, optimal velocity models, and cellular automata models (Brackstone and McDonald, 1999).

Collision avoidance or safety distance models are based on the fundamental principle that a following vehicle maintains a safe distance from the leader to avoid collisions (Pipes, 1953; Forbes, 1963; Kometani and Sasaki, 1958; Gipps, 1981; Krauss, 1997). Stimulus-response models were created under the assumption that the driver of the following vehicle accurately perceives and reacts appropriately to the spacing and speed differences between following and leading vehicles (Gazis et al., 1959; Newell, 1961; Herman and Rothery, 1965). Psycho-physical or action point models offer a more realistic representation of driving behaviour by accounting for imperfect perception and discontinuous responses based on thresholds related to stimuli like visual angle, speed, spacing, and more (Lee, 1976; Wiedemann, 1974; Fritzsche, 1994). Optimal velocity or desired measures models are based on the assumption that the follower will accelerate or decelerate depending on the difference between their actual driving measures and the desired optimum or target measures (Helly, 1961; Bando et al., 1995).

The Cellular Automata (CA) model, initially developed by Nagel and Schreckenberg (1992), is widely recognized as the NaSch model, where cells are considered as fundamental units updated at each time step. Other well-known microscopic models prevalent in the literature include the Intelligent Driver Model (Bando et al., 1995), which considers continuous car-following, the Das and Asundi model that relates vehicular speed to vehicular density (Das and Asundi, 2012), the Van Aerde model (Aerde and Rakha, 1995), applicable to all traffic states, and the fuzzy logic model (Chakroborty and Kikuchi, 2003), which predicts a range of possible reactions using fuzzy membership functions.

General Motors (GM) conducted extensive and comprehensive field experiments that played a pivotal role in bridging the gap between microscopic and macroscopic modelling approaches, ultimately leading to the development of response-stimulus models (Gazis et al., 1959). Numerous studies have been conducted to enhance the GM model by incorporating various aspects of following behaviour. For instance, Edie (1961) extended the model's applicability to scenarios with less-than-optimal traffic densities, considering variations in driver sensitivity based on absolute speed. May and Keller (1967) derived a set of speed and spacing coefficients tailored to different traffic situations. Heyes and Ashworth (1972) introduced a steady-state single-regime model at the aggregate level. Additionally, the evaluation of single and dual-regime traffic flow models for congested and uncongested conditions led to the development of spacing and speed exponents, as explored by Ceder and May (1976). To address distinct acceleration and deceleration regimes, modifications were made to the GM model structure (Ahmed, 1999). Choudhury and Islam (2016) considered multiple sources of stimulus in non-lane-based traffic conditions using a latent leader acceleration model. Gao et al. (2018) put forth a novel LRVD model (Left and Right Lane Velocity Difference Model), taking into account relative speeds on different lanes. It's worth noting that a majority of these models either focused on modelling or calibrated vehicular interactions using homogeneous lane-based data.

Mixed traffic conditions introduce various factors such as off-centered following (Gunay 2007), lateral shifting (Mahapatra et al., 2018), staggered following (Madhu et al., 2020), influence area concept (Madhu et al., 2020a), driving regimes (Madhu et al., 2022) and adjacent vehicle influence (Madhu et al., 2023). Various approaches, like the porous flow method, have been employed to simulate the zig-zag movement of vehicles in mixed traffic, particularly the interaction between two-wheelers and passenger cars (Nair et al., 2011). Further advancements include the division of road space into strips and the development of a space discretization-based simulation framework for mixed traffic by Mathew et al. (2015). Researchers like Asaithambi et al. (2018) have evaluated different vehicle-following models (e.g., Gipps, Intelligent Driver Model, Krauss Model, and Das and Asundi) within microscopic traffic simulation models for mid-block sections.

Examining multiple leader scenarios, Budhkar and Maurya (2017) considered the effects of average speed and center-line separation distance on longitudinal gap maintenance in mixed traffic. In another approach, Kanagaraj and Treiber (2018) used disordered self-driven particle systems to model mixed traffic conditions and proposed a generalized multi-particle model. Papathanasopoulou and Antoniou (2018) developed data-driven models for mixed traffic using trajectory data from India. They classified vehicle following behaviours into different manoeuvres based on leader-follower pair combinations and lateral gaps (Madhu et al., 2020). Kashyap et al. (2020) employed oblique trajectories and relative speed hysteresis plots to identify vehicle pairs in the steady-state following regime, while Sharath and Velaga (2020) modelled two-dimensional (lateral and longitudinal) vehicle motion using the Intelligent Driver Model (IDM). Das et al. (2020) emphasized the manoeuvrability of vehicles in mixed traffic conditions and its impact on traffic flow. Notably, the diverse static and dynamic characteristics of vehicles underscore the significance of surrounding vehicles in modelling following behaviour in mixed traffic (Madhu et al. 2020a). There are studies that have explored the role of surrounding vehicles in vehicular movements in mixed traffic (Raju et al., 2021; Patil et al., 2021). Paul et al. (2021) investigated road network safety and efficiency by segregating smaller vehicles in mixed traffic conditions. Madhu et al. (2023) examined the influence of adjacent vehicles and their various orientations on the driving behaviour of the subject vehicle.

A review of the literature indicates that very few studies have ventured into modelling and calibrating vehicular interactions based on empirical data in mixed traffic scenarios. Additionally, there is a notable absence of studies exploring the influence of adjacent vehicles and their spatial configurations in the literature. Consequently, from literature survey, notable research gap is identified in examining and analysing the influence of surrounding vehicles on subject vehicle driving decisions, which plays a pivotal role in persuasively defining heterogeneous traffic condition. Of the different neighbouring vehicles present, the acceleration and lane changing decisions are primarily dependent on the leaders. From the literature survey, it is evident that attempts to identify multiple leader orientation and to consider it as the stimulus to analyse the response of subject vehicle from vehicle trajectory data are limited. To fill these gaps, present study develops a model structure for vehicle following in mixed traffic by incorporating multiple leader dynamic attributes. These developed models are used to investigate the effect of sensitivity and stimulus of cars and two-wheelers (TW) towards primary leader across multiple leader spatial orientations.

3. Methodology

The methodology adopted in this study is illustrated in Figure 1. The steps involve: trajectory data collection, subject and surrounding vehicle demarcation, manoeuvre identification, single and multiple leader categorizations, model formulation and analysis. The scope of the study is limited to the following behaviour of vehicles in an urban midblock section.

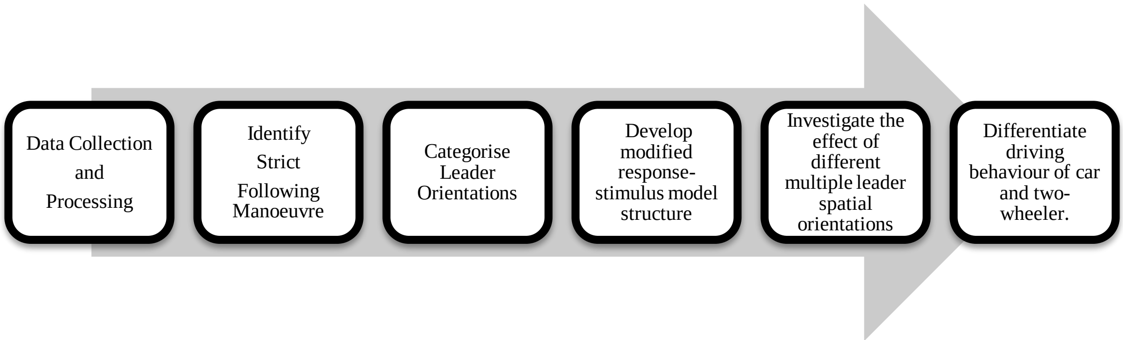


Figure 1: Methodology Adopted for Analysing the Role of Multiple Leaders and in Its Dynamic Variables in Following Behaviour of Subject Vehicle

The vehicles in the study stretch are divided into the subject vehicle and surrounding vehicles. The time varying response of subject vehicle is assumed to be dependent on the actions of the surrounding vehicles. Of the different surrounding vehicles, this study considers the leaders, their spatial orientation and its dynamic variables in modelling the longitudinal response of the subject vehicle. The trajectory data for each vehicle is extracted using semi-automated methods. Manual data collection methods are time-consuming and labour-intensive. Conversely, fully automated methods in mixed traffic conditions may lead to incorrect vehicle identification and partial trajectory extraction. Therefore, Python's graphical user interface (GUI) was utilized for continuous vehicle tracking (Madhu et al., 2020). The position, orientation, and attributes of multiple leaders in relation to the subject vehicle are determined based on the concept of the influence area. The influence area is defined as the region surrounding the subject vehicle where other vehicles are present and can impact the driving behaviour of the subject vehicle (Madhu et al., 2020a). Acceleration models are formulated for different spatial orientations of leaders by incorporating the attributes of multiple leaders. These models are used to evaluate the driving behaviour of vehicles in mixed traffic condition.

4. Vehicle Trajectory Data Collection and Processing

The primary requirement of any microscopic traffic flow modelling is the availability of accurate vehicle level movement and trajectory data. An extensive reconnaissance survey was carried out in selecting the appropriate mid-block section, camera location, viewing angle, etc. The site was chosen such that maximum coverage of road section was possible with a safe placement of camera, so that vehicles will be either moving away/towards the camera. A semi-automated data collection strategy was established using Python 2.7 graphical user interface (Madhu et al., 2020). The data was collected on a Wednesday (which is a weekday and also a working day). Friday and Monday were avoided since they will be influenced by weekend traffic fluctuations. On the chosen day, grid lines were marked on the selected road section during the off-peak hour for camera calibration. The corners of the grids were marked and were video recorded. The trajectories of vehicles were extracted using Python's graphical user interface, where the mouse click of each vehicle coordinate is related to the frame number to capture the time-varying position coordinates. This helps in tracking each vehicle with a unique vehicle ID in the study stretch from entry to exit point with the timestamp recorded from frame number and spatial coordinates from the corresponding mouse clicks. The recorded pixel coordinates of spatial points were converted to ground coordinates using a camera calibration technique (Madhu et al., 2020). The primary spatial data was converted to secondary data of speed and acceleration of the vehicle. Thus, the final data contains the vehicle ID, vehicle type, its length and width along with position, speed, and acceleration along the longitudinal and lateral direction corresponding to each instant of time. This data was analysed in MATLAB to identify the subject vehicle along with the corresponding positions of surrounding vehicles. From the leader-follower combination and the extent of overlap between them, the vehicle following manoeuvres were classified into strict, staggered and non-overlap following, and the analysis was done separately for each manoeuvre type.

The vehicle positions coordinates are given as (x_i, y_i) at each instant of time t_i (1 second). The total length of study stretch is 250 m having a road width of 10.5 m, which consists of three lanes in one direction, separated by a median. To assess the precision of the extracted trajectory, the position coordinate in the longitudinal direction is predicted using the synthesised speed and acceleration values, using Equation 1, and is compared with actual position coordinates.

$$x_{i+1} = x_i + [v_i^x t + \frac{1}{2} a_i^x t^2] \tag{1}$$

where, $x_i + 1$ is the predicted position coordinate at the $(i + 1)^{th}$ instant, x_i is the actual position coordinate extracted from vehicle trajectory video at i^{th} instant, v_i^x and a_i^x are the synthesised values of speed and acceleration.

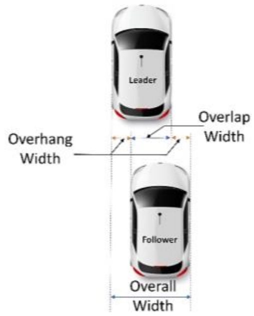
The Mean Absolute Percentage Error (MAPE) value indicating the difference between the actual and predicted values is found to be 0.554%, which can be considered as sufficiently accurate. Thus, it can be concluded that the vehicle trajectory extraction methodology followed in this study is accurate.

5. Definition of Terms and Exploratory Analysis

The definitions of new terms and their explanations are provided within this section. This section covers the major aspects of mixed traffic features, including various vehicle following manoeuvres and their identification. Additionally, this section also discusses the identification and categorization of multiple leader orientations.

5.1 Vehicle Following Manoeuvres Identification and Classification

Three categories of following have been observed and identified from the vehicle trajectory data: strict, staggered, and non-overlap following. Non-overlap following occurs when the lateral dimensions of the leader and subject vehicle do not overlap, while overlapping following is when the subject vehicle and leader's lateral dimensions (widths) overlap with each other, as shown in Figure 2. This can be further classified into strict and staggered vehicle-following manoeuvres based on lateral offset (centre to centre separation) between leader and follower pair. Smaller lateral offset indicates considerable overlap between lead-lag pair, resulting in strict following manoeuvre. Larger lateral offset between lead-lag pair for overlap following case indicates staggered following manoeuvre.



(Overlap width is the lateral dimension of the follower overlapping with the leader and overall width is the width between the extreme ends of leader and follower)

Figure 2: Pictorial Representation of Overlap Width, Overhang Width and Overall Width under Following Manoeuvres

The lateral offset value between the leader and follower is set to ensure a significant overlap between the two vehicles. In mixed traffic conditions, the smallest possible vehicle pair combination involves a two-wheeler (TW) and an auto-rickshaw (Auto). When considering a TW-Auto pair, the minimum central separation possible, whether both are right or left-aligned, is 40 cm. Therefore, for a conservative approach in defining strict following, the laterally overlapping lead-lag pair is considered strictly following if the central separation is less than 40 cm; otherwise, it is classified as staggered. The longitudinal gap maintained between lead-lag vehicle pairs in three different manoeuvre conditions is compared and depicted in the Figure 3.

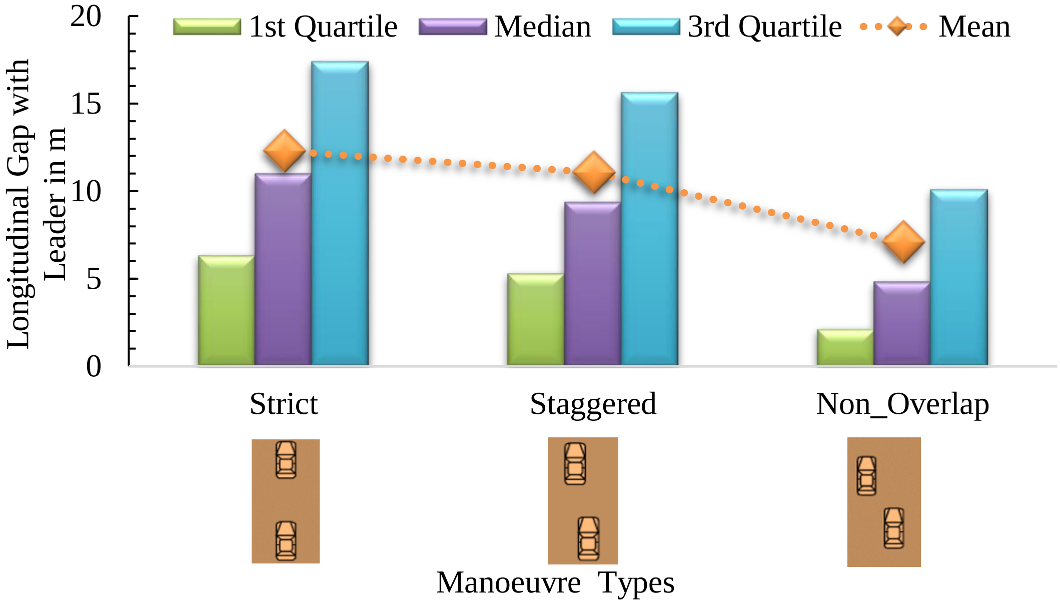
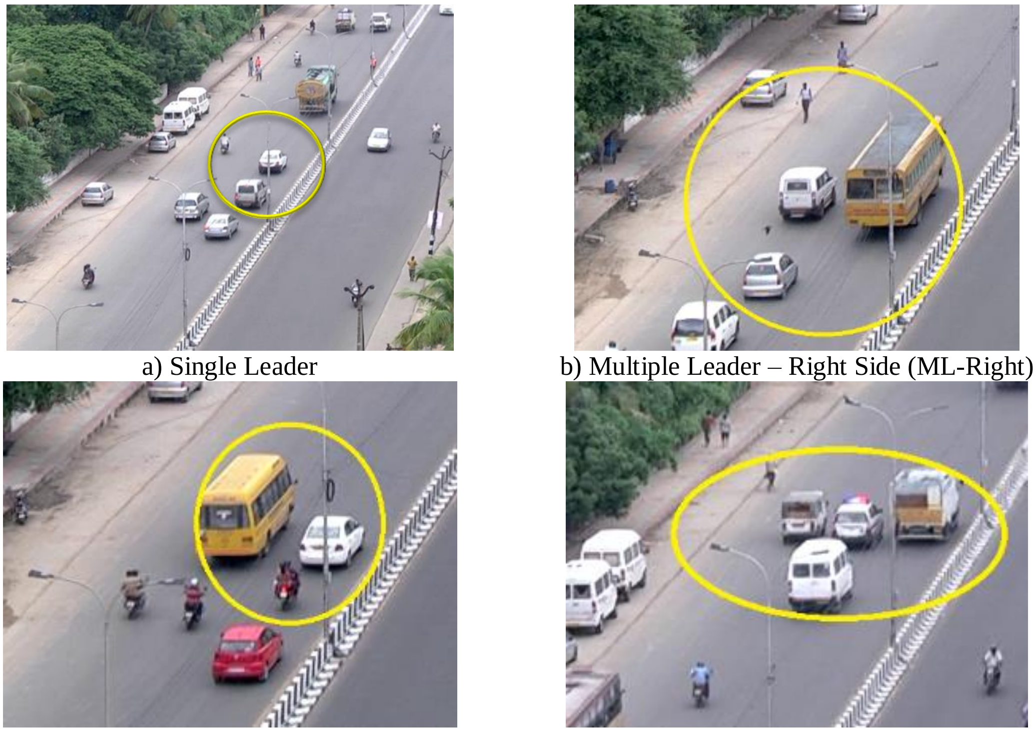


Figure 3: Variation in Longitudinal Gap with respect to Different Manoeuvres

It has been observed that the longitudinal gap is at its maximum during the strict following manoeuvre and at its minimum during the non-overlap manoeuvre. The longitudinal gap maintained during the staggered manoeuvre falls between the two extremes. In strict following, the lateral offset between the leader and follower is minimal, resulting in a larger maintained longitudinal gap. Conversely, in the staggered following manoeuvre, the higher lateral offset between the leader and follower leads to a smaller longitudinal gap, as it allows for more lateral freedom. In the non-overlap manoeuvre, the follower is not confined within the leader's path, granting it maximum freedom for both longitudinal and lateral movement, thus maintaining a minimal longitudinal gap with the leader. Consequently, as one progresses from strict following to non-overlap following, the longitudinal gap between the leader and follower diminishes while the lateral separation increases.

5.2 Criteria for Demarcating Single and Multiple Leaders.

Leader combinations can be classified into a single leader and multiple leaders with different spatial orientations as shown in Figure 4.



a) Single Leader

b) Multiple Leader – Right Side (ML-Right)

c) Multiple Leader – Left Side (ML-Left)

d) Multiple Leader – Both Sides (ML-Both)

Figure 4: Single and Multiple Leader Orientations

The spatial arrangement of primary and subsidiary leaders with respect to the subject vehicle results in various combinations of multiple leaders. The primary leader is identified as the one engaged in a strict following manoeuvre. Once the strict or primary leader is identified, other vehicles located in front, which are longitudinally overlapping with the primary leader, are identified and referred to as subsidiary leaders. If there is only one leader present in front, it is categorized as a single leader (SL) as shown in Figure 4a. Multiple Leader-Right (ML: Right) as in Figure 4b represents non-overlapping subsidiary leader present to the right-hand side of the primary leader, whereas Multiple Leader-Left (ML: Left) as in Figure 4c represents the placement of non-overlapping subsidiary leader to the left-hand side of the primary leader. Figure 4d displays Multiple leaders-Both sides (ML: Both), the condition with three leaders, with one being the primary leader and the other two being subsidiary leaders. The leader combination distribution from the field data with the strictly overlapping primary leader is shown in Figure 5.

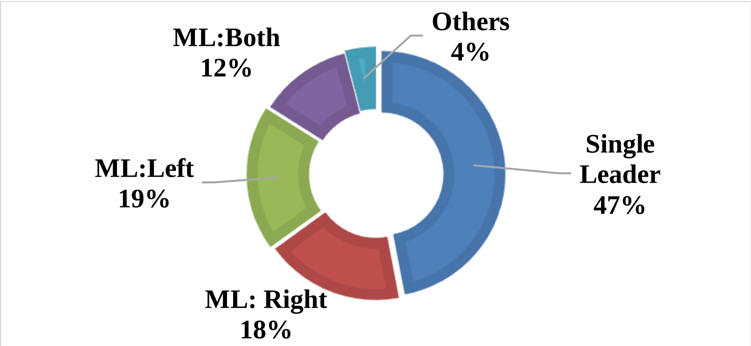


Figure 5: Distribution of Multiple Leader Orientations under Strict Following Manoeuvres

Single leader cases account for 47% of the total, while multiple leader instances make up the remaining 53%. The significant prevalence of multiple leaders underscores the need for a distinct examination of such cases in mixed traffic conditions. Among the multiple leader cases, those involving a subject vehicle with two leaders (either ML: Right or ML: Left) constitute 37%, while cases with three leaders (ML-Both) amount to 12%. All other possible leader combinations fall into a miscellaneous category, which collectively comprises less than 5% of the dataset. Due to sample size limitations, this miscellaneous category has not been included in the analysis. The model framework for both single and multiple leaders will be discussed in the following section.

6. Modified response-stimulus model structure for Single and Multiple Leaders

Car-following models simulate the motion of individual vehicles through mathematical models by considering the safety aspect, thus incorporating collision avoidance with leading vehicles in the stream. The acceleration, or rate of change of speed is regarded as the response of the particle, and this response is dependent on the stimulus it receives from the environment. The General Motors (GM) model considered relative speed between the leader and follower as the stimulus. However, the reaction of the follower is dependent not only on the stimulus, but also on the sensitivity of a vehicle to the stimulus it receives. This sensitivity may be composed of the absolute speed of the subject vehicle, spacing with the leader, etc. Therefore, the acceleration can be written as a function of these three variables, as represented in Equation 2.

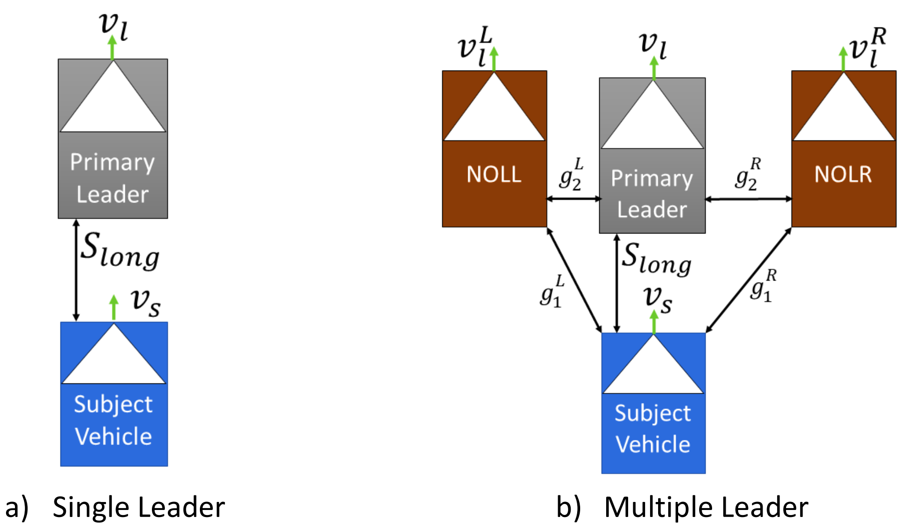
$$a_s(t+\tau) = K \frac{v_s^{\alpha}(t)}{S_{long}^{\beta}(t)} [v_l(t) - v_s(t)]^{\gamma} \tag{2}$$

where, $a_s\,(t + \tau)$ is the longitudinal response of the subject vehicle after reaction time τ, which can be either acceleration or deceleration, $[v_l(t) - v_s(t)]^{\gamma}$ is the stimulus which is the relative speed between the leader and subject vehicle (follower) at a given time t, γ is sensitivity towards relative speed, $v_s^{\alpha}(t)$ is the absolute speed of the subject vehicle at time t, α is the sensitivity towards subject vehicle's speed, $S_{long}^{\beta}(t)$ is the longitudinal gap between the leader and follower at time t and β is the sensitivity towards the gap.

The above model can be converted into a linear regression form by taking logarithms of both LHS and RHS of Equation 2. In addition to this model, several other structural forms of stimulus-response model are evaluated in order to select the best model based on performance measures and behavioural realism. The following models were evaluated on the dataset of nearly 13014 observations – (a) Combined linear stimulus-response model, (b) separate linear stimulus-response model for acceleration and deceleration phases, (c) log-log transformed model, (d) box-cox transformed model, (e) power law models. The goodness of fit measures varied across these models and interestingly the simplest model (combined linear stimulus-response model) turns out to have better explanatory power and easier to interpret than the more complex model structures. Hence the analysis in the subsequent sections is based on this model structure as given in Equation 3.

$$a_s(t+\tau) = K + \alpha v_s(t) - \beta S_{long}(t) + \gamma(v_l(t) - v_s(t)) \tag{3}$$

Equation 3 is modified for both single and multiple leader conditions in the following sections. The schematic representations of single and multiple leaders, along with their dynamic variables are depicted in Figure 6a and Figure 6b, respectively.



(v_l is the speed of the primary leader and v_s is the speed of the subject vehicle, NOLL and NOLR are left and right-side subsidiary leaders with respect to the subject vehicle, L and R are used to represent the parameters of left and right-side leaders, respectively, g_2 is the lateral gap between the subsidiary and primary leaders, g_1 is the diagonal gap of subject vehicle with the subsidiary leader.)

Figure 6: Layout of Single Leader and Generalised Multiple Leader Orientations

6.1 Stimulus-Response Model for a Single Leader.

For a single leader (as shown in Figure 6a), the subject vehicle is receiving stimulus from only the single leader and hence the response is directly proportional to stimulus and the sensitivity it receives from that leader. The longitudinal response model given in Equation 4 is applied only to single leader scenarios and the coefficients are calibrated separately.

$$a_s(t+\tau) = K_{SL} + \alpha_{SL} v_s(t) - \beta_{SL} S_{long}(t) + \gamma_{SL}(v_l(t) - v_s(t)) \quad (4)$$

where, the subscript SL represents the coefficients of single leader

6.2 Modified Stimulus-response Model with Multiple Leader Dynamic Variables.

To incorporate the weak lane discipline condition of mixed traffic into the stimulus-response model, the concept of multiple leaders have been introduced. For multiple leaders, the response model should effectively capture the effect of subsidiary leaders present in front. The dynamic variables of multiple leaders (as given in Figure 6b) used for modelling include:

a) Diagonal (oblique) gap of a subsidiary leader with subject vehicle (g_1)
b) Lateral gap between primary and subsidiary leaders(g_2)
c) Relative speed between primary and subsidiary leaders (ΔV_L).

The presence of multiple leaders (ML) can be incorporated by integrating the multiple leader attribute into sensitivity and stimulus terms in Equation 3. The present study considers dynamic variables of multiple leaders (ML) and investigates how it influences the sensitivity and stimulus of a follower towards the leader.

The factors that differentiate the response of a follower to multiple leaders as compared to a single leader can be explained through the terms g_1, g_2 and ΔV_L. For multiple leaders, the stimulus is the relative speed between the primary leader and subject vehicle and its interaction with g_1, g_2 and ΔV_L . The sensitivity terms considered are absolute speed of the subject vehicle and the longitudinal gap between leader and subject vehicle and its interaction with g_1, g_2 and ΔV_L. Different interaction combinations of multiple leader dynamic variables with base variables (absolute speed, longitudinal gap, and relative speed) are done using probabilistic statistical measures that attempt to quantify both the model performance on the training dataset and the complexity of the model. The best model form is selected by comparing the Akaike information criterion (AIC), Bayesian information criterion (BIC) and adjusted R^2 values. The model form with ML dynamic variable interactions with intercept, longitudinal gap and relative speed is finally selected having minimum AIC and BIC values with maximum adjusted R^2. Thus, the interaction of ML-parameters with the sensitivity (intercept and longitudinal gap) and stimulus (relative speed) terms in Equation 3 is carried out and the modified model is given in Equation 5.

$$a_s(t+\tau) = K_{ML} + \alpha v_s(t) - \beta_{ML} S_{long}(t) + \gamma_{ML}(v_l(t) - v_s(t)) \quad (5)$$

where,

$$K_{ML} = K_0 + K_1 g_1^L + K_2 g_2^L + K_3 \delta |v_l^L - v_l| + K_4 g_1^R + K_5 g_2^R + K_6 \delta |v_l^R - v_l| \quad (5a)$$

$$\beta_{ML} = \beta_0 + \beta_1 g_1^L + \beta_2 g_2^L + \beta_3 \delta |v_l^L - v_l| + \beta_4 g_1^R + \beta_5 g_2^R + \beta_6 \delta |v_l^R - v_l| \quad (5b)$$

$$\gamma_{ML} = \gamma_0 + \gamma_1 g_1^L + \gamma_2 g_2^L + \gamma_3 \delta |v_l^L - v_l| + \gamma_4 g_1^R + \gamma_5 g_2^R + \gamma_6 \delta |v_l^R - v_l| \quad (5c)$$

where, the subscript ML represents the multiple leader orientation coefficients, and l is the primary leader, superscript L and R represents parameters of left and right side subsidiary leaders, respectively, g_1 is the oblique gap of subject vehicle with the subsidiary leader, g_2 is the lateral gap between the subsidiary and primary leaders, δ is a dummy variable indicating positive speed difference between the subsidiary and primary leaders; if the speed difference between them is positive, then δ is 1, otherwise 0.

7. Variation in Driving Behaviour of Car and Two-wheeler Based on Dynamic Variables of Multiple Leader Spatial Orientations

A comparison of modified model (Equation 5 *with ML dynamic variables*) and base model (Equation 3) is done to evaluate whether the addition of independent variables could improve the model. An F-test (Gujarati, 2004) is performed to compare between the restricted (base) model with the unrestricted (modified) model using the Equation 6. The F-statistic formula calculates how much of the variance in the dependent variable, the base model is not able to explain as compared to the modified model, which is expressed as a fraction of the unexplained variance from the modified model. The corresponding fit measures indicate an F_{stat}=15.43 against F_{table}=1.61 for alpha = 5% significance level, implying that the modified model (with ML dynamic variables) to be superior to base model.

$$F_{statistic} = \frac{\left(\frac{RSS_{base} - RSS_{modified}}{k_{modified} - k_{base}}\right)}{\left(\frac{RSS_{modified}}{n - k_{modified}}\right)} \tag{6}$$

where, RSS_{base} and $RSS_{modified}$ are the residual sum of squares of base restricted and modified unrestricted models, respectively, k_{base} and $k_{modified}$ are the number of estimated parameters in the restricted and unrestricted models, respectively, and n is the total number of data samples.

The model estimates are given in Table 2 to Table 5 which tabulate the interaction of explanatory variables with intercept, absolute speed, longitudinal gap, relative speed, respectively, for categories ML-right, ML-left and ML-both of multiple leader cases.

7.1 Goodness of fit measures.

Table 1 presents the goodness of fit of the estimated models. The goodness of fit has considerably improved from base to modified stimulus-response models for multiple leader cases. The coefficient of multiple determination, R^2 has improved, and the mean absolute error (MAE) has reduced. Further, segmenting the data based on car and TW yield models with better predictability and realism.

Table 1 Goodness of fit and Sample Size of Stimulus-Response Modified Model

Segmented Models for Multiple Leaders		Sample Size	R^2	MAE
Aggregate		5905	0.532	0.872
ML_Right	TW	2344	0.524	0.889
	Car	321	0.488	0.790
ML _Left	TW	1777	0.555	0.904
	Car	776	0.568	0.761
ML _Both	TW	572	0.553	0.862
	Car	115	0.656	0.634

7.2 Effect of Intercept.

The intercept in the stimulus-response model typically represents the acceleration/deceleration of subject vehicle due to unidentified parameters. Comparison of intercept across models is done based on the coefficient given in Table 2. The intercept value is highest for both TW and car under ML-left orientation. The intercept for TW is minimum for ML-right. On the other hand, the intercept for car is negative in ML-both indicating deceleration of car in the presence of both side multiple leaders. This shows that car in the presence of multiple leaders on both sides gets constrained by the movement. These differences in coefficients arise due to the effect of ML dynamic variables.

For TW being the subject vehicle, having subsidiary leader on the right side, the positive speed difference between subsidiary and primary leader (coefficient 0.115) is significant and other variables are insignificant. For car being subject vehicle, with right side subsidiary leader, the intercept is not influenced by any of the variables (lateral or oblique gap, and speed difference among leaders).

Table 2 Intercept and it's Interaction with Multiple Leader Dynamic Variables

Segmented Models for Multiple Leaders		Intercept, ***K***						
		k_0	k_1 (g_1^L)	k_2 (g_2^L)	k_3 (ΔV_l^L)	k_4 (g_1^R)	k_5 (g_2^R)	k_6 (ΔV_l^R)
Aggregate		0.521 (2.95)			0.0749 (3.41)	0.0211 (2.98)		0.125 (3.14)
ML-Right	TW	0.630 (3.17)	NA	NA	NA			0.115 (3.25)
	Car	0.041 (0.08)	NA	NA	NA			
ML-Left	TW	1.081 (4.99)		0.105 (2.29)	0.088 (1.49)	NA	NA	NA
	Car	0.847 (2.56)	0.027 (1.48)		0.174 (2.27)	NA	NA	NA
ML-Both	TW	0.841 (1.98)	0.067 (1.47)	0.194 (1.48)				0.126 (1.80)
	Car	-1.119 (-1.44)						0.129 (1.65)

- t values are given in parenthesis
- k estimate corresponding to population parameter *K*

If the subsidiary leader is only on the left side, then the lateral gap has a positive effect (0.106) for two-wheeler, whereas the oblique gap has a positive effect (0.027) for the car. For both TW and car as the following vehicle, the speed difference among leaders has a positive influence and is more for car (0.174) than for two-wheeler (0.0882). With multiple leaders on both sides, two-wheelers are seen to accelerate with increasing lateral (0.194) and oblique gap (0.067) on the left. Both two-wheelers and cars are found to accelerate (0.125, 0.129) when the right-side leader is faster than the front leader.

7.3 Effect of subject vehicle speed.

In Table 3 the influence of adjacent vehicles on sensitivity to speed is considered. The coefficient of speed is higher for the ML-left case than right case for both two-wheeler (-0.101 and -0.086) and car (-0.074 and -0.049). Also, two-wheelers appear to be slightly more sensitive to their own speed than cars. The speed for ML-both is statistically insignificant for car but is significant for TW (coefficient is -0.089). The nature of error in the predicted value by the aggregate model varies with leader combinations and subject vehicle types.

Table 3 Absolute Speed Coefficient Estimate of Modified Model

Segmented Models for Multiple Leaders		a ($\boldsymbol{Absolute\ speed, v_s}$)
Aggregate		-0.083 (-6.63)
ML-Right	TW	-0.086 (-6.65)
	Car	-0.049 (-1.50)
ML-Left	TW	-0.101 (-7.14)
	Car	-0.074 (-3.40)
ML-Both	TW	-0.089 (-2.98)
	Car	-0.041 (-0.63)

- t values are given in parenthesis
- a parameter estimate corresponding to population parameter α

7.4 Effect of Longitudinal Gap.

The effect of multiple leader dynamic attributes on the sensitivity of follower to longitudinal gap with leader is examined in Table 4. Note that the coefficient b0 represents the influence of longitudinal gap with front leader on follower's acceleration behaviour, when lateral gap between front subsidiary leader is near to zero, and the primary and subsidiary leaders are travelling at the same speed. This coefficient has a negative sign indicating a more cautious behaviour (higher deceleration rates) at smaller gaps.

Table 4 Longitudinal Gap and its Interaction with ML Dynamic Variables

Segmented Models for Multiple Leaders		Longitudinal Gap, $\boldsymbol{\beta}$						
		b_0	b_1 (g_1^L)	b_2 (g_2^L)	b_3 (ΔV_l^L)	b_4 (g_1^R)	b_5 (g_2^R)	b_6 (ΔV_l^R)
Aggregate		-0.023 (-2.92)				0.001 (2.26)	0.007 (1.48)	0.003 (1.80)
ML-Right	TW	-0.049 (-3.02)	NA	NA	NA	0.001 (2.53)	0.006 (1.52)	0.004 (1.96)
	Car	-0.026 (-0.62)	NA	NA	NA			
ML-Left	TW	-0.012 (-0.69)			0.005 (1.69)	NA	NA	NA
	Car	-0.039 (-1.63)				NA	NA	NA
ML-Both	TW	-0.015 (-0.34)		0.018 (1.92)			0.015 (1.69)	
	Car	-0.118 (-1.97)	0.005 (1.89)			0.009 (1.70)		0.016 (1.92)

- t values are given in parenthesis
- b estimate corresponding to population parameter β

For a subsidiary leader only on the right side (ML-right), TW's show a smaller sensitivity to gap with front leader as the lateral gap (0.0061), oblique gap (0.0012) or speed difference (0.0037) between right leader and front leader increases. This suggests that when there is sufficient gap between front and right leader, or right leader and following vehicle, two-wheeler will not decelerate to the same extent as it would otherwise. Similarly, when the right-side vehicle is accelerating compared to the front vehicle, the TW becomes less sensitive to gap with front vehicle. This suggest that two-wheelers are more likely to adopt an opportunistic gap-seeking driving behaviour than a strictly following one.

The following behaviour with regard to longitudinal gap sensitivity is quite different when the subsidiary leader is only on the left-hand side. TW's responsiveness to gap is affected only by the speed difference among the leaders (0.0048), but not the lateral gap between leaders or oblique gap between it and the left side leader. This is consistent with a general tendency to overtake only on the right. However, in both cases (one side subsidiary leader ML-right or ML-left), the sensitivity of cars to longitudinal gap with leader is not influenced by either gap or speed difference among leaders. The reason for this is probably due to its larger size and lesser opportunities for overtaking than two-wheelers.

In contrast, lateral restraint on both sides (ML-both) affects following behaviour of both two-wheelers and cars but in different ways. The coefficient of longitudinal gap reduces for two-wheelers as the lateral gap on either side (left: 0.0179 or right: 0.0149) between front leader and subsidiary leaders increases. This implies that for a given longitudinal gap, the acceleration of two-wheelers will be more when the lateral gap is wider on either side. A similar effect is seen for cars but with respect to oblique gap between it and the subsidiary leader (0.0089 and 0.0052 on right and left side), as it needs more space during lateral shifting manoeuvres. As the oblique gap between subject vehicle and subsidiary leader increases the sensitivity to front leader gap reduces. Cars also show a greater acceleration propensity for a given spacing as the speed difference between right side leader and front leader increases (0.0162). As the right-side leader is faster than the front leader, the sensitivity towards gap with front leader reduces. But speed difference on the left has no effect. This is consistent with tendency to shift to right if a gap is created by a faster subsidiary leader or can lead to increased speed for the following vehicle.

7.5 Effect of relative speed.

The relative speed variations across ML orientations and the effect of dynamic variables are examined in Table 5. The first column c_0 indicates the effect of relative speed difference (when the effect of other variables are constant). Across ML orientations, the coefficient of relative speed with leader is maximum for ML-both, followed by ML-left and minimum for ML-right. This shows that if the subject vehicle's movement to the left or right is restricted by the presence of subsidiary leaders on both sides, the effect of stimulus received from leader increases.

When subsidiary leader is present only on either left or right, the relative speed difference among leaders affects the coefficient of relative speed for both two-wheeler and cars. The relative speed coefficient decreases by 0.0116, 0.0273, 0.0209,0.0295 as speed difference between subsidiary and front leader increases by 1 m/s for TW-ML-right, car-ML-right, TW-ML-left, and car-ML-left cases respectively. Thus, the influence of speed difference with front leader weakens when the side leader is faster.

If the subsidiary leader is present on only one side, the oblique gaps (and not lateral gaps) affect the relative speed coefficients for both two-wheelers and cars. The oblique gap coefficients are -0.012 on the right side and -0.011 on the left side for two-wheeler, whereas this coefficient is only significant for cars on the left side and slightly larger

(-0.019). Thus, larger gaps between subject vehicle and subsidiary vehicle (on only one side) reduces the effect of relative speed (with front vehicle).

Table 5 Relative Speed and its Interaction with ML Dynamic Variables

Segmented Models for Multiple Leaders		Relative Speed, γ						
		c_0	c_1 (g_1^L)	c_2 (g_2^L)	c_3 (ΔV_l^L)	c_4 (g_1^R)	c_5 (g_2^R)	c_6 (ΔV_l^R)
Aggregate		0.742 (3.40)	-0.011 (-2.26)	0.022 (3.21)		-0.009 (-1.78)	-0.025 (-1.98)	0.015 (1.82)
ML-Right	TW	0.624 (2.94)	NA	NA	NA	-0.012 (-1.78)		-0.012 (-1.82)
	Car	0.520 (6.57)	NA	NA	NA			-0.027 (-1.61)
ML - Left	TW	0.749 (17.85)	-0.011 (-1.76)		-0.021 (-2.10)	NA	NA	NA
	Car	0.767 (13.39)	-0.019 (-7.54)		-0.029 (-2.01)	NA	NA	NA
ML - Both	TW	0.840 (2.76)	-0.013 (-1.87)	-0.046 (-1.99)				
	Car	0.887 (2.44)						-0.101 (-1.5)

- t values are given in parenthesis
- c parameter estimate corresponding to population parameter γ

However, in cases where the subsidiary leader is present on both sides, the sensitivity of cars to relative speed is affected only by the speed difference between the right subsidiary leader and front leader (-0.101) which is a reflection of looking out for shifting or overtaking opportunities on the right side. However, for two-wheelers, the left side lateral gap (-0.046) and oblique gap (-0.013) appear to reduce the sensitivity to relative speed, indicating its propensity to shift on left side also when constrained by leaders in front, left and right.

Comparing the effect of spatial orientation of multiple leaders, both qualitative and quantitative differences are observed in acceleration response of subject vehicle based on spatial orientation.

8. Conclusions

Various transformations of the stimulus-response model for mixed traffic data have been explored, and multiple linear regression was chosen due to its simplicity in prediction and interpretation, along with better explanatory power. Modifications in the multiple leader model, compared to a single leader, were made by allowing different sensitivity (intercept) and slopes for relative speed, gap, and speed of the following vehicle. The results demonstrate that in mixed traffic conditions, relying solely on the stimulus-response following theory with a single leader is insufficient. It requires careful integration of multiple leader attributes.

This study highlights the significance of multiple leaders and their attributes in explaining vehicle following behaviour. In addition to the influence of the longitudinal gap with the primary leader, the oblique gap between the subject vehicle and side leaders, as well as the lateral gap between front and side leaders, also impact the following response. Multiple leader attributes affect following behaviour in mixed traffic in various ways, as noted above. These effects vary based on the number of leaders and the location of subsidiary leaders. Additionally, the lateral gap and speed difference between subsidiary and front leaders affect the following behaviour of the subject vehicle in certain cases. The oblique gap between the subject vehicle and subsidiary leader alters the influence of relative speed between the front leader and the follower. These effects also differ between two-wheelers and cars. Consequently, this analysis demonstrates that sensitivity to relative speed and gap with the primary leader decreases with increased lateral gap, speed difference between leaders, and oblique gap with the subject vehicle and subsidiary leader. Neglecting these effects results in statistically inferior models and erroneous estimates of relative speed, longitudinal gap, or absolute speed of the subject vehicle while following.

Significant differences in driving behaviour between cars and two-wheelers are evident when considering various multiple leader orientations. Two-wheelers exhibit a slightly higher sensitivity to their own speed compared to cars. In the presence of multiple leaders on both sides, cars experience constraints on their movement, resulting in deceleration compared to two-wheelers. The study also suggests that two-wheelers are more inclined toward opportunistic gap-seeking driving behaviours rather than strictly adhering to a following behaviour, as is often seen with cars. Car responses are more influenced by the gap between leaders compared to two-wheelers, possibly due to their larger size and fewer opportunities for overtaking compared to two-wheelers. Cars also tend to accelerate more when there is a greater speed difference between the right-side leader and the front leader. As the right-side leader's speed exceeds that of the front leader, sensitivity to the gap with the front leader decreases. This behaviour aligns with the tendency of cars to shift to the right when a gap is created by a faster subsidiary leader, potentially leading to increased speed for the following vehicle. In contrast, for two-wheelers, the left side lateral gap and oblique gap appear to reduce sensitivity to relative speed, indicating a propensity to shift to the left when constrained by leaders in front, left, and right.

The study concludes that enhancing the stimulus-response model with multiple leader attributes improves the predictability of acceleration models and enables the capture of complex interactions between the subject vehicle, primary leader, and subsidiary leaders. These models have potential applications in traffic simulation with proper calibration and validation, aiding in quantifying factors such as capacity, level of service, and surrogate safety measures.

The multiple leader car-following model represents a significant advancement in the field of traffic modelling and simulation. This model extends the understanding of vehicle interactions in mixed traffic conditions, where vehicles of various types and sizes share the road with differing driving behaviours. By considering the influence of multiple leading vehicles on a following vehicle, this model captures the complexity of real-world traffic scenarios more accurately. One of its key benefits is improved predictive power, allowing for more precise estimations of vehicle behaviours, including acceleration, deceleration, and lane-changing manoeuvres. Moreover, the multiple leader car-following model offers insights into the role of surrounding vehicles, helping to identify patterns and factors that affect traffic flow and safety. As a result, it has valuable applications in traffic management, transportation planning, and the development of advanced driver assistance systems, contributing to safer and more efficient road networks.

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