



Cross-docks scheduling with multiple doors using fuzzy approach

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Abstract

One of the most important practices in logistics is Cross-Docking which sets its goals as inventory reduction and customer satisfaction increase. Customers receive goods through docks. Docks are responsible to provide a place for goods before being delivered to the customers. Then, these materials are directly loaded into outbound trucks with little or no storage in between to send to customers in the shortest possible time. This paper is mainly aimed at introducing a mixed-integer, non-linear programming model to solve scheduling several cross-docking problems. The proposed model is highly facilitated to allocate the most optimal destinations to storage doors and truck scheduling in docks while selecting the collection and delivery routes. Using optimization approaches at uncertainty conditions is also of great importance. Mathematical programming techniques vividly fail to solve transportation problems that include fuzzy objective function coefficients. A fuzzy multi-objective linear programming model is proposed to solve the transportation decision-making with fuzzy objective function coefficients.

Keywords: Cross Docks Scheduling, Fuzzy Logic, supply chain

1. Introduction

Many companies use Cross-docking as a logistic strategy to ensure storage costs reduction and customer satisfaction improvement within a shorter delivery lead-time. Space requirements, inventory warehousing costs, labor-intensive, and order picking tasks can be mentioned among the main reasons behind the high expense of goods storage. Cross-docking is the true tool to eliminate a large portion of such warehousing costs. A cross-dock is defined as an I-shaped facility with strip and stack dock doors located at opposite sides of the terminal and minimal storage space in between. Strip docks located on one side of the distribution terminal receive inbound shipments arriving at the cross-dock. As soon as the inbound trucks are unloaded, the freights are screened and sorted by destination. Then, a forklift or a conveyor belt is used to move them across the terminal via their designated stack dock doors, and here is where the loads are charged into departing trucks to be carried to their destinations. Since workers are responsible for unloading, sorting, and transferring a wide variety

of loads from incoming trucks to outgoing trailers, freight handling is vividly considered as a labor-intensive and costly task in a cross-dock terminal.

Products with the best matching to cross-docking include (a) products with a stable demand; (b) perishable bulk materials, consist of some chemical and food compounds, requiring immediate shipment; (c) frozen foods and other refrigerated products like pharmaceuticals that should be directly moved from cooled inbound to cooled outbound trucks to keep the cooling chain unbroken; (d) high-quality items of low-quality inspection requirements during the receiving process and (e) ready pre-tagged products for being sold to the customers. Furthermore, hazardous chemicals drums and waste materials containers are aggregated at cross-dock facilities and immediately transferred to remedy sites for treatment and disposal. Cross-docking is regarded as a tool for pharmaceutical, food, and chemical industries day in day out to achieve more competitive advantages. In a practical approach, successful implementation of cross-docking strategies is evident in chemical and manufacturing companies such as Eastman Kodak Co., Goodyear GB Ltd. and Toyota [1]. Comprehensive reviews on cross-docking were also offered by [1] and [2].

Both location and physical layout of a cross-dock facility have been subject to many types of research so far regarding shape and number of dock doors, and related truck scheduling. Routing aspects of the problem were neglected, though. Operational issues at the cross-dock terminal are considered in the truck scheduling (TS) problem and are mainly addressed in assigning vehicles to dock doors, the processing sequence of trucks at every strip and stack door, and transferring goods from inbound to outbound vehicles. The presence of temporary storage is always necessary despite the fact that cross-docking is supposed to unload inbound trucks and immediately reload the freights into delivery vehicles. The absence and impossibility of a perfect synchronization in limited numbers of pickup and delivery trucks lead goods to fail in arriving at the cross-dock in the sequence they must be reloaded into the departing vehicles.

In the mentioned cross-dock operation planning and scheduling models, we assumed several docks and uncertainty of parameters as one of the decisions making challenges. Not only, an efficient scheduling model is provided, but also, finding an optimal approach in the presence of uncertainty is of great importance.

2. literature review

Table (1). Articles on cross-dock truck scheduling

Title	Research work	Objective function	Solution method
“Cross-docking-JIT scheduling with time windows”	Li, Y., Lim, A., Rodrigues, B. (2004)	Minimize storage and order picking activity	Genetic Algorithm
“Multiple cross docks with inventory and time windows”	Chen, P., Guo, Y., Lim, A., Rodrigues, B. (2006)	Minimize the cost distribution	Simulated annealing and tabu search heuristics
“Vehicle routing scheduling for cross-docking in the supply chain”	Lee, Y.H., Jung, J.W., Lee, K.M. (2006)	Minimize the transportation cost	Tabu search algorithm

Title	Research work	Objective function	Solution method
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“Scheduling of inbound and outbound trucks in cross-docking systems with temporary storage”	Yu, W., Egbelu, P. J. (2008)	Minimize total operation time	Heuristic algorithm
“Vehicle routing with cross-docking”	Wen, M., Larsen, J., Clausen, J., Cordeau, J.F., Laporte, G. (2009)	Minimize the total travel time	Tabu search algorithm
“Truck dock assignment problem with operational time constraint within cross docks”	Miao, Z., Lim, A., Ma, H. (2009)	Minimize the makespan	Simulated annealing algorithm (SA)
“Meta-heuristics implementation for scheduling of trucks in a cross-docking system with temporary storage”	Boloori Arabani, A.R., FatemiGhomi, S.M., Zandieh, M. (2011)	Minimize the total operation time	Genetic algorithm (GA), tabu search (TS), particle swarm optimization (PSO), ant colony optimization (ACO) and differential evolution (DE)
“A Sweep-Heuristic based formulation for the vehicle routing problem with cross-docking”	Dondo, R., Cerdá, J. (2012)	Minimize the makespan	Sweep heuristic algorithm
“A sequential priority-based heuristic for scheduling material handling in a satellite cross-dock”	Maknoon, M.Y., Kone, O., Baptiste, P. (2014)	Maximizes the total number of products that are transferred directly	The sequential priority-based heuristic algorithm
“On-line cross-docking: A general new concept at a container port”	Azimi, P. (2015)	Minimize the average annual system costs	Hybrid meta-heuristic algorithm based on Genetic Algorithm (GA) and the simulation technique
“A bi-objective truck scheduling problem in a cross-docking center with a probability of breakdown for trucks”	Amini, A., Tavakkoli-Moghaddam, R. (2016)	Minimize the total tardiness of outbound trucks	Multi-objective meta-heuristics: Non-dominated Sorting Genetic Algorithm II (NSGA-II), MultiObjective Simulated Annealing (MOSA) and Multi-Objective Differential Evolutionary (MODE)
“An improved mathematical model and a hybrid metaheuristic for truck scheduling in cross-dock problems”	Keshtzari, M., Naderi, B., Mehdizadeh, E. (2016)	Minimize total operational time or makespan	Particle swarm optimization hybridized with a simulated annealing
“A location-routing problem for cross-docking networks: A biogeography-based optimization algorithm”	Hasani Goodarzi, A., Zegordi, S. H. (2016)	Minimize the total cost vehicles	A metaheuristic algorithm named Biogeography-based optimization (BBO)
“Differential evolution and Population-based simulated annealing for truck scheduling problem in multiple door cross-docking systems”	Assadi, M. T., Bagheri, M. (2016)	Minimize total earliness and tardiness for outbound trucks, in such systems	Two meta-heuristics called Differential evolution and Population-based simulated annealing
“Truck scheduling in multi-door cross-docking terminal by modified particle swarm optimization”	Wisittipanich, W., Hengmeechai, P. (2017)	Minimize total operational time or makespan	Extension of Particle Swarm Optimization (PSO), named GLNPSO, with an aim to improve the performance of the original PSO
Title	Research work	Objective function	Solution method

“Scheduling of truck arrivals, truck departures and shop-floor operation in a cross-dock platform, based on trucks loading plans”	Serrano, C., Delorme, X., Dolgui, A. (2017)	Minimize penalty costs	The proposed model was implemented and tested with CPLEX
“Scheduling of loading and unloading operations in a multi stations transshipment terminal with release date and inventory constraints”	Bazgosha, A., Ranjbar, M., Jamili, N. (2017)	Minimizes the makespan	Develop two constructive heuristic solution approaches, namely parallel and serial schedule generation schemes
“Truck scheduling in cross-docking systems with fixed due dates and shipment sorting”	Molavi, D., Shahmardan, A., Sajadieh, M. S. (2018)	Minimize the weighted sum of delayed shipments as well as the delivery cost of remained shipments	Genetic algorithm (GA), differential evolution (DE) and particle swarm optimization (PSO)
“Capacitated and multiple cross-docked vehicle routing problems with pickup, delivery, and time windows”	Ahkamiraad, A., Wang, Y. (2018)	Minimize the transportation cost and the fixed cost in a specified planning horizon	Hybrid of the genetic algorithm and particle swarm optimization (HGP)
“Optimizing the number of outbound doors in the cross-dock based on a new queuing system with the assumption of beta arrival time”	Motaghedi-Larjani, A., Aminnayeri, M. (2018)	Minimize the total costs, including the costs of adding a new outbound door and the expected waiting time of customers	Conditional probability

To achieve this goal, we presented a new monolithic MILP formulation that integrates the pickup/delivery vehicle routing and scheduling with both the assignment of dock-doors to incoming and outgoing trucks and the managing of truck queues at strip/stack doors. Attracted to the surveys of Dondo and Cerdá [21], we avoided symmetrical solutions by embedding additional constraints imitating the sweeping algorithm to develop an efficient hybrid approach capable of solving medium-size problem instances at acceptable CPU times. In the mentioned cross-dock operation planning and scheduling models, we assumed several docks and uncertainty of parameters as one of the decisions making challenges. Not only, an efficient scheduling model is provided, but also, finding an optimal approach in the presence of uncertainty is of great importance. Mathematical programming techniques and equations have proved their disabilities in solving transportation decision-making problems by fuzzy objective function coefficients. To overcome these problems, we provided a fuzzy-interactive multi-objective linear programming model for solving transportation decision problems by fuzzy objective function coefficients in the present research and concluded its computational flexibility and efficiency, at the end.

3. Problem description and formulation

VRPCD-TS problem which is defined as a combinational vehicle routing and cross-dock truck scheduling problem focus on transporting a set of requests R from pickup to destination points passing through an intermediate cross-dock facility at minimum routing cost. A limited number of receiving (strip) doors RD and shipping (stack) doors SD are assumed in the cross-dock. In order to increase the cross-dock

productivity and reduce the handling cost, the dock door to which an inbound (outbound) truck arrives (departs) at (from) the cross-dock, is determined from the very beginning. The truck scheduling (TS) problem seeks to find the optimal assignment of inbound/outbound trucks to dock doors. The majority of the studies on the VRPCD problem consider the same number of dock doors and trucks, so each truck will be assigned to a different door and truck scheduling aspects can be ignored. However, if this condition is not met, the dock doors will be seen as scarce resources that need to be scheduled overtime, and lines of trucks waiting for service can arise at every dock door and this is the real so-called truck scheduling problem. A sequential manner is offered for VRPCD and truck scheduling (TS) problems simultaneous solving because of their complexities. However, we didn't categorize this combinational problem into two phases assuming a limited number of dock doors compared to other studies in this scope.

Different from Dondo and Cerdá [21] that studied vehicle routing and scheduling problem by a cross-dock, we modeled the vehicle routing and scheduling problem using several cross docks in this paper. Due to the lack of data completeness and availability, the decision-making of cross docks operation scheduling and planning generally face inaccurate data as well as transport planning [22]. One of the inevitable challenges that we encounter while making decisions on cross-dock operation scheduling and planning problems is the uncertainty of parameters. Therefore, there should be proper approaches to lead us to optimized solutions in uncertain situations besides the presence of an efficient timing model. Parameters such as costs, demand, and production capacity are very likely to be uncertain in the cross-dock scheduling problem [23]. Supply chain planning researches mainly focus on potential distribution relying on previous data to address uncertainties. Probable models may not be the best choice because of the lack of availability and also the reliability of the previous statistical data [26]. On the contrary, fuzzy set and possibility theories proved to be superior to probability theories in facing an uncertain supply chain, besides their simplicity and no requirement for data collection [24]. Baykasoglu and Göçken [25] classified fuzzy mathematical programming problems detecting 15 different types of fuzzy mathematical programming models and provided different solution approaches for each type. Mathematical programming techniques and equations proved not to be capable of solving transportation decision-making problems by fuzzy objective function coefficients. To get rid of this deficiency and respect computational flexibility and efficiency, we proposed a fuzzy-interactive multi-objective linear programming model for solving transportation decision problems by fuzzy objective function coefficients.

3.1. Problem assumptions

Inspired by Dondo and Cerdá [21] the mathematical formulation has been developed based on the following assumptions.

1. Goods are transported from suppliers to destinations by a homogeneous vehicle fleet through a single cross-dock terminal.
2. The well-known layout for cross-dock includes a specific number of strip and stack dock doors.
3. At the beginning of the planning horizon, all vehicles are assumed to be available, accomplish the required pickup tasks, and subsequently perform the delivery tasks.

4. Dock doors are exclusively dedicated to either unloading or loading operations, e.g. they are designated as either strip or stack dock doors.
5. The number of strip/stack doors can be lower than the number of vehicles. Then, the dock doors can be regarded as scarce resources that should be scheduled overtime.
6. Each P/D request must be serviced by a single vehicle, i.e. orders are not splittable.
7. The loading/unloading of a truck at the cross-dock cannot be interrupted, i.e. no pre-emption is allowed.
8. The freights unloaded at the cross-dock are not interchangeable, i.e. each one must be sent to a specific destination.
9. The amounts of loaded or unloaded goods at supply/delivery locations are given.
10. Each vehicle is allowed to service more than one pick-up/delivery location.
11. The starting and ending point for the pickup and delivery routes are set to the cross-dock.
12. The total quantity of goods carried by a vehicle must not exceed its capacity.
13. The sum of a fixed stop time (ft_r^P/ft_r^D) and a variable component determines the service time at supply/delivery locations and is increased with the size of the cargo q_r to be picked-up/delivered at a rate l_r/u_r .
14. The goods picked up and delivered by the same truck are not unloaded at the cross-dock and remain inside the vehicle.
15. The total amount of goods unloaded on the receiving docks and the total freight loaded on trucks at the shipping doors must be equal at the end of the planning horizon. Therefore, there is no final inventory left at the cross-dock.

Sets:

N : unload events

R : requests

RD : receiving (strip) dock doors

SD : shipping (stack) dock doors

V : vehicles

W : cross-docks

Parameters:

To generalize the proposed model by Dondo and Cerdá [21], we defined the following parameters cross docks capacity.

$\widetilde{d}_{r,f}^P/\widetilde{d}_{r,f}^D$: the distance between P/D locations r and f

$\widetilde{d}_{r,w}^P/\widetilde{d}_{r,w}^D$: the distance between the P/D location r and the cross-dock w

$\widetilde{ft}_r^P/\widetilde{ft}_r^D$: fixed stop time at the P/D site of request r

$\widetilde{ft}_w^P/\widetilde{ft}_w^D$: fixed stop time for P/D activities at the cross-dock terminal w

$\widetilde{l}_r/\widetilde{u}_r$: loading/unloading rate at P/D sites of request r

$\widetilde{l}_w/\widetilde{l}_w$: loading/unloading rate at the cross-dock terminal w

\widetilde{q}_r : shipment size for request r

\widetilde{Q}_v : vehicle capacity

\widetilde{Q}_w : cross-dock capacity

\widetilde{sp}_v : vehicle travel speed

$\widetilde{tt}_{d,\acute{d}}$: time spent in moving a vehicle from the unloading door $d \in RD$ to the shipping door $\acute{d} \in SD$

\widetilde{uc}_v : unit distance cost for vehicle v

μ : cost per unit of spent time for the accomplishment of delivery and loading tasks.

Binary variables:

We also defined binary variables of $GP_{w,v}/GD_{w,v}$ for modeling the allocation of vehicles to cross-dock as another aspect to generalize Dondo and Cerda's [21] model.

$DP_{v,d}/DD_{v,d}$: denotes that vehicle v has been allocated to the strip/stack dock door d

$WP_{n,v}/WD_{n,v}$: denotes that the unloading(U)/loading(L) activity of vehicle v is associated to the time event n .

$XP_{r,f}/XD_{r,f}$: establishes the sequencing of pickup(P)/delivery(D) nodes (r, f) on the route of the assigned P/D vehicle

$YP_{r,v} / YD_{r,v}$: denotes that vehicle v visits the P/D location of request r

$ZP_{v,\hat{v}}/ZD_{v,\hat{v}}$: sequences vehicles (v, \hat{v}) waiting for service at the same strip/stack door

$GP_{w,v}/GD_{w,v}$: denotes that vehicle v visits the P/D location of cross-dock w

Nonnegative continuous variables:

AT_v^P/AT_v^D : P/D vehicle arrival times of vehicle v at the cross-dock facility

CP_r/CD_r : Cumulative traveling cost from the cross-dock to the P/D site of request r

$DRS_{v,d,\hat{d}}$: denotes that the receiving door $d \in RD$ and the shipping door $\hat{d} \in SD$ have been assigned to vehicle v

OC_v^P/OC_v^D : overall traveling cost for the P/D tour of vehicle v

RT_v^P : time at which vehicle v is released from its pickup duties

ST_v^P/ST_v^D : starting time for the P/D tour of vehicle v

TP_r/TD_r : vehicle arrival time at the P/D node of request r

TE_n : unload time-event n

$UR_{r,n,v}$: denotes that request r was unloaded from vehicle v before or exactly at time TE_n

$UT_{r,n}$: denotes that the request was unloaded on the cross-dock before or exactly at time event n

$YR_{r,v}$: states that the P/D locations of request r are both served by vehicle v

3.2. Problem formulation

Considering fuzzy parameters, the mathematical model of the problem was calculated as follows:

$$\text{Min } z_1 = \sum_{v \in V} [(OC_v^P + OC_v^D)] \quad (1)$$

$$\text{Min } z_2 = \sum_{v \in V} AT_v^D \quad (2)$$

$$\text{Min } z_3 = \mu \sum_{v \in V} AT_v^D + \sum_{v \in V} [(OC_v^P + OC_v^D)] \quad (3)$$

s.t.

$$\sum_{v \in V} GP_{w,v} = 1; \forall w \in W \quad (4)$$

$$\sum_{v \in V} YP_{r,v} = 1; \forall r \in R \quad (5)$$

$$CP_r \geq \widetilde{uc}_v d_{w,r}^P YP_{r,v} GP_{w,v}; \forall r \in R, v \in V, w \in W \quad (6)$$

$$CP_f \geq CP_r + \widetilde{uc}_v d_{r,f}^P - M_C^P(1 - XP_{r,f}) - M_C^P(2 - YP_{r,v} - YP_{f,v}); \forall r, f \in R (r < f), v \in V \quad (7)$$

$$CP_r \geq CP_f + \widetilde{uc}_v d_{r,f}^P - M_C^P XP_{r,f} - M_C^P(2 - YP_{r,v} - YP_{f,v}); \forall r, f \in R (f < r), v \in V \quad (8)$$

$$OC_v^P \geq CP_r + \widetilde{uc}_v d_{r,w}^P - M_C^P(1 - YP_{r,v}); \forall r \in R, v \in V, w \in W \quad (9)$$

$$TP_r \geq ST_v^P + \left(\frac{d_{w,r}^P}{sp}\right) YP_{r,v} GP_{w,v}; \forall r \in R, v \in V, w \in W \quad (10)$$

$$TP_f \geq TP_r + \widetilde{ft}_r^P + \widetilde{lr}_r \widetilde{qr} + \left(\frac{d_{r,f}^P}{sp}\right) - M_T^P(1 - XP_{r,f}) - M_T^P(2 - YP_{r,v} - YP_{f,v}) \quad (11)$$

; $\forall r, f \in R (r < f), v \in V$

$$TP_r \geq TP_f + \widetilde{ft}_f^P + \widetilde{lr}_f \widetilde{qr} + \left(\frac{d_{r,f}^P}{sp}\right) - M_T^P XP_{r,f} - M_T^P(2 - YP_{r,v} - YP_{f,v}) \quad (12)$$

; $\forall r, f \in R (f < r), v \in V$

$$AT_v^P \geq TP_r + \widetilde{ft}_r^P + \widetilde{lr}_r \widetilde{qr} + \left(\frac{d_{r,w}^P}{sp}\right) - M_T^P(1 - YP_{r,v}); \forall r \in R, v \in V, w \in W \quad (13)$$

$$\sum_{r \in R} \widetilde{qr} YP_{r,v} \leq \widetilde{Q}_v; \forall v \in V \quad (14)$$

$$\sum_{r \in R} \widetilde{qr} YP_{r,v} GP_{w,v} \leq \widetilde{Q}_w; \forall v \in V, w \in W \quad (15)$$

$$YR_{r,v} \leq YP_{r,v}; \forall r \in R, v \in V \quad (16)$$

$$YR_{r,v} \leq YD_{r,v}; \forall r \in R, v \in V \quad (17)$$

$$YR_{r,v} \geq YP_{r,v} + YD_{r,v} - 1; \forall r \in R, v \in V \quad (18)$$

$$\sum_{d \in RD} DP_{v,d} = 1; \forall v \in V \quad (19)$$

$$RT_v^P \geq AT_v^P + \widetilde{ft}_w^P + \widetilde{ur}_w GP_{w,v} [\sum_{r \in R} \widetilde{qr} (YP_{r,v} - YR_{r,v})]; \forall r \in R, v \in V, w \in W \quad (20)$$

$$RT_{v'}^P \geq RT_v^P + \widetilde{ft}_w^P + \widetilde{ur}_w GP_{w,v'} [\sum_{r \in R} \widetilde{qr} (YP_{r,v'} - YR_{r,v'})] - M_T^P(1 - ZP_{v,v'}) - M_T^P(2 - DP_{v,d} - DP_{v',d}); \forall d \in RD, v, v' \in V (v < v'), w \in W \quad (21)$$

$$RT_v^P \geq RT_{v'}^P + \widetilde{ft}_w^P + \widetilde{ur}_w GP_{w,v} [\sum_{r \in R} \widetilde{qr} (YP_{r,v} - YR_{r,v})] - M_T^P(1 - ZP_{v,v'}) - M_T^P(2 - DP_{v,d} - DP_{v',d}); \forall d \in RD, v, v' \in V (v' < v), w \in W \quad (22)$$

$$\sum_{n \in N} WP_{n,v} = 1; \forall v \in V \quad (23)$$

$$\sum_{v \in V} WP_{n,v} = 1; \forall n \in N \quad (24)$$

$$TE_{\acute{n}} \geq TE_n; \forall n, \acute{n} \in N (n < \acute{n}) \quad (25)$$

$$TE_{\acute{n}} \geq RT_v^P + M_T^P(WP_{n,v} - 1); \forall n, \acute{n} \in N (n < \acute{n}), v \in V \quad (26)$$

$$RT_v^P \leq TE_{\acute{n}} + M_T^P(1 - WP_{n,v}); \forall n, \acute{n} \in N (n < \acute{n}), v \in V \quad (27)$$

$$\sum_{n \in N} TE_n = \sum_{v \in V} RT_v^P \quad (28)$$

$$TE_n \leq RT_v^P; \forall n = \text{first}(N), v \in V \quad (29)$$

$$TE_n \geq RT_v^P; \forall n = \text{last}(N), v \in V \quad (30)$$

$$UR_{r,n,v} \leq WP_{n,v}; \forall n \in N, r \in R, v \in V \quad (31)$$

$$\sum_{n \in N} UR_{r,n,v} \leq YP_{r,v}; \forall r \in R, v \in V \quad (32)$$

$$UR_{r,n,v} \geq (WP_{n,v} + YP_{r,v} - 1); \forall n \in N, r \in R, v \in V \quad (33)$$

$$UT_{r,n} = \sum_{\acute{n} \in N} \sum_{v \in V} UR_{r,\acute{n},v}; \forall n \in N, r \in R \quad (34)$$

$$ZP_{v,v'} \leq 2 - WP_{n,v} - \sum_{\acute{n} < n} WP_{\acute{n},v'}; \forall n \in N, v, v' \in V (v < v') \quad (35)$$

$$ZP_{v,v'} \geq WP_{n,v} + \sum_{\acute{n} > n} WP_{\acute{n},v'} - 1; \forall n \in N, v, v' \in V (v < v') \quad (36)$$

$$\sum_{v \in V} GD_{w,v} = 1; \forall w \in W \quad (37)$$

$$\sum_{v \in V} YD_{r,v} = 1; \forall r \in R \quad (38)$$

$$\sum_{\acute{d} \in SD} DD_{v,\acute{d}} = 1; \forall v \in V \quad (39)$$

$$DRS_{v,d,\acute{d}} \leq DP_{v,d}; \forall v \in V, d \in RD, \acute{d} \in SD \quad (40)$$

$$DRS_{v,d,\acute{d}} \leq DD_{v,\acute{d}}; \forall v \in V, d \in RD, \acute{d} \in SD \quad (41)$$

$$DRS_{v,d,\acute{d}} \geq DP_{v,d} + DD_{v,\acute{d}} - 1; \forall v \in V, d \in RD, \acute{d} \in SD \quad (42)$$

$$\sum_{d \in RD} \sum_{\acute{d} \in SD} DRS_{v,d,\acute{d}} = 1; \forall v \in V \quad (43)$$

$$ST_v^D \geq RT_v^P + \sum_{d \in RD} \sum_{\acute{d} \in SD} \widetilde{tt}_{d,\acute{d}} DRS_{v,d,\acute{d}} + \widetilde{ft}_w^D + \widetilde{ur}_w GD_{w,v} [\sum_{r \in R} \widetilde{qr} (YD_{r,v} - YR_{r,v})] \quad (44)$$

; $\forall v \in V, w \in W$

$$ST_{v'}^D \geq ST_v^D + \sum_{d \in RD} \sum_{\acute{d} \in SD} \widetilde{tt}_{d,\acute{d}} DRS_{v',d,\acute{d}} + \widetilde{ft}_w^D + \widetilde{ur}_w GD_{w,v'} [\sum_{r \in R} \widetilde{qr} (YD_{r,v'} - YR_{r,v'})] - M_T^D(1 - ZD_{v,v'}) - M_T^D(2 - DD_{v,\acute{d}} - DD_{v',\acute{d}}); \forall v \in V, w \in W \quad (45)$$

$$ST_v^D \geq ST_{v'}^D + \sum_{d \in RD} \sum_{\acute{d} \in SD} \widetilde{tt}_{d,\acute{d}} DRS_{v,d,\acute{d}} + \widetilde{ft}_w^D + \widetilde{ur}_w GD_{w,v} [\sum_{r \in R} \widetilde{qr} (YD_{r,v} - YR_{r,v})] - M_T^D ZD_{v,v'} - M_T^D(2 - DD_{v,\acute{d}} - DD_{v',\acute{d}}); \forall v \in V, w \in W \quad (46)$$

$$\sum_{n \in N} WD_{n,v} = 1; \forall v \in V \quad (47)$$

$$UT_{r,n} \geq (WD_{n,v} + YD_{r,v} - 1); \forall n \in N, r \in R, v \in V \quad (48)$$

$$ST_v^D \geq TE_n + \sum_{d \in RD} \sum_{\acute{d} \in SD} \widetilde{tt}_{d,\acute{d}} DRS_{v,d,\acute{d}} + \widetilde{ft}_w^D + \widetilde{ur}_w GD_{w,v} [\sum_{r \in R} \widetilde{qr} (YD_{r,v} - YR_{r,v})] - M_T^D(1 - WD_{n,v}); \forall n \in N, v \in V, w \in W \quad (49)$$

$$WD_{n,v} \leq \sum_{\substack{n \in N \\ n \leq n}} WP_{n,v} \quad ; \forall n \in N, v \in V \quad (50)$$

$$CD_r \geq \widehat{uc}_v d_{w,r}^D YD_{r,v} GD_{w,v} \quad ; \forall r \in R, v \in V, w \in W \quad (51)$$

$$CD_f \geq CD_r + \widehat{uc}_v d_{r,f}^D - M_C^D (1 - XD_{r,f}) - M_C^D (2 - YD_{r,v} - YD_{f,v}) \quad (52)$$

$; \forall r, f \in R (r < f), v \in V$

$$CD_r \geq CD_f + \widehat{uc}_v d_{r,f}^D - M_C^D XD_{r,f} - M_C^D (2 - YD_{r,v} - YD_{f,v}) \quad ; \forall r, f \in R (f < r), v \in V \quad (53)$$

$$OC_v^D \geq CD_r + \widehat{uc}_v d_{r,w}^D - M_C^D (1 - YD_{r,v}) \quad ; \forall r \in R, v \in V, w \in W \quad (54)$$

$$TD_r \geq ST_v^D + \left(\frac{d_{w,r}^D}{sp} \right) YD_{r,v} GD_{w,v} \quad ; \forall r \in R, v \in V, w \in W \quad (55)$$

$$TD_f \geq TD_r + \widehat{ft}_r^D + \widehat{ur}_r \widehat{qr} + \left(\frac{d_{r,f}^D}{sp} \right) - M_T^D (1 - XP_{r,f}) - M_T^{PD} (2 - YD_{r,v} - YD_{f,v}) \quad (56)$$

$; \forall r, f \in R (r < f), v \in V$

$$TD_r \geq TD_f + \widehat{ft}_f^D + \widehat{ur}_f \widehat{qr} + \left(\frac{d_{r,f}^D}{sp} \right) - M_T^D XP_{r,f} - M_T^D (2 - YD_{r,v} - YD_{f,v}) \quad (57)$$

$; \forall r, f \in R (f < r), v \in V$

$$AT_v^D \geq TD_r + \widehat{ft}_r^D + \widehat{ur}_r \widehat{qr} + \left(\frac{d_{r,w}^D}{sp} \right) - M_T^D (1 - YD_{r,v}) \quad ; \forall r \in R, v \in V, w \in W \quad (58)$$

$$\sum_{r \in R} \widehat{qr} YD_{r,v} \leq \widehat{Q}_v \quad ; \forall v \in V \quad (59)$$

$$\sum_{r \in R} \widehat{qr} YD_{r,v} GD_{w,v} \leq \widehat{Q}_w \quad ; \forall v \in V, w \in W \quad (60)$$

$$ZD_{v,\acute{v}} \leq 2 - WD_{n,v} - \sum_{\substack{n \in N \\ n < n}} WD_{n,\acute{v}} \quad ; \forall n \in N, v, \acute{v} \in V (v < \acute{v}) \quad (61)$$

$$ZD_{v,\acute{v}} \geq WD_{n,v} + \sum_{\substack{n \in N \\ n > n}} WD_{n,\acute{v}} - 1 \quad ; \forall n \in N, v, \acute{v} \in V (v < \acute{v}) \quad (62)$$

$$AT_v^P \geq (1 - \eta_P) \left[ST_v^P + \left(\frac{OC_v^P}{uc_v sp} \right) + \sum_{r \in R} (\widehat{ft}_r^P + \widehat{lr}_r \widehat{qr}) YP_{r,v} \right] \quad ; \forall v \in V \quad (63)$$

$$AT_v^P \leq (1 + \eta_P) \left[ST_v^P + \left(\frac{OC_v^P}{uc_v sp} \right) + \sum_{r \in R} (\widehat{ft}_r^P + \widehat{lr}_r \widehat{qr}) YP_{r,v} \right] \quad ; \forall v \in V \quad (64)$$

$$AT_v^D \geq (1 - \eta_D) \left[ST_v^D + \left(\frac{OC_v^D}{uc_v sp} \right) + \sum_{r \in R} (\widehat{ft}_r^D + \widehat{lr}_r \widehat{qr}) YD_{r,v} \right] \quad ; \forall v \in V \quad (65)$$

$$AT_v^D \leq (1 + \eta_D) \left[ST_v^D + \left(\frac{OC_v^D}{uc_v sp} \right) + \sum_{r \in R} (\widehat{ft}_r^D + \widehat{lr}_r \widehat{qr}) YD_{r,v} \right] \quad ; \forall v \in V \quad (66)$$

$$\sum_{\substack{d \in RD \\ d < n}} DP_{v,d} \geq \sum_{\substack{n \in N \\ n < n}} WP_{n,v} \quad ; \forall v \in V, n \in N (n \leq |RD|) \quad (67)$$

Objective function z_1 tries to minimize cumulative routing cost of the vehicle; and objective function z_2 aims at minimizing cumulative distribution time. Objective function z_3 seeks to minimize the coordinated composition of the first and second objectives. It should be noted that coefficient μ indicates costs per unit time spent on the accomplishment of delivery and loading tasks in the third objective function.

Eq.(4) refers to the allocation of loading vehicles to docks. Each vehicle should be allocated to a dock. If the allocation variable $GP_{w,v}$ is equal to 1, vehicle v will serve dock w . Eq. (5) assigns requests to pick up vehicles. The pickup location of each request must be allocated to a single-vehicle.

The pickup node of request r will be served by the inbound vehicle v provided that the assignment variable $YP_{r,v}$ is equal to 1. Eq.(6) defines the routing cost from the cross-dock up to the first visited node on a pickup route and provides a lower bound on the routing cost from the cross-dock to any pickup node served by vehicle v , including the first visited location. The parameter uc_v represents the routing cost per unit distance and $d_{w,r}^D$ denotes the distance between the cross-docks, identified by the sub script w , and the pickup site of request r .

Both constraints (7) and (8) try to relate the cumulative routing costs from the cross-docks to the pickup sites of a pair of requests ($r, f \in R$) served by the same vehicle v (i.e. $YP_{r,v} = YP_{f,v} = 1$). A single binary variable $XP_{r,f}$ (with $r < f$) to select the relative order of any pair of pick-up nodes (r, f) located on the same inbound route in this formulation. If $XP_{r,f} = 1$ ($r < f$), then the request r is served earlier than

\acute{r} . By Eq. (7), therefore, $CP_{\acute{r}}$ must be larger than CP_r by at least the routing cost along the path directly connecting both locations, i.e. the shortest route between the pickup sites of r and \acute{r} . Otherwise, $XP_{r,\acute{r}} = 0$ and node \acute{r} are seen before node r . Consequently, $CP_{\acute{r}}$ should be lower than CP_r by at least the cost term $(uc_v d_{r,\acute{r}}^P)$ which is met by Eq.(8). It is worth mentioning that parameter M_C^P is a relatively large number.

Eq.(9) indicates the overall routing cost for the tour allocated to pickup vehicle v . each pickup route should end at the cross-dock facility. Since there is an unknown string of nodes on the route before solving the model, Eq. (9) provides a lower bound on the total routing cost for the vehicle tour OC_v^P considering any node on the route as the last visited one. The value of OC_v^P determined by the largest bound is set by the pickup location that is actually last visited by vehicle v . pickup node visiting times and vehicle arrival times at the cross-docks are presented in Eqs (10)-(13). These equations provide the opportunity to determine both the visiting time for the pickup location r (TP_r) and the v th-vehicle arrival time (AT_v^P) at the cross-dock. Vehicle v should wait its turn on the queue of the assigned strip dock door till the end of unloading operations. The timing constraints (11)-(12) present the same mathematical structures as Eqs (7)-(8). These sequencing constraints consider routing time parameters instead of routing cost coefficients. The service time at any pickup node r is the sum of two terms: a fixed preparation time ft_r^P plus the variable loading time that directly increases with the load size q_r . The proportionality constant lr_r stands for the loading rate at the pickup node r . Moreover, the routing time along the path connecting the pickup nodes r and \acute{r} is given by the ratio between the distance $d_{r,\acute{r}}^P$ and the vehicle speed sp_v . If all pickup routes are started at time $t = 0$, then $ST_v^P = 0$ for all $v \in V$, the continuous variable ST_v^P stands for the starting time of the v th-pickup route.

Eq. (14) doesn't allow the load transported by vehicle v to exceed its maximum capacity (Q_v). Eq.(15) doesn't allow the load transported by cross-dock w cannot exceed its maximum capacity (Q_w).

Eqs (16)-(18) represent pickup node visiting times and vehicle arrival times at the cross-docks. When pickup and delivery sites of the request r are both served by the same vehicle, the related transshipment operations at the cross-dock are not required. In such a case, $YP_{r,v} = YD_{r,v} = 1$ for some vehicle v and the load of request r is not discharged on the receiving dock, i.e. it remains into the vehicle v . define $YR_{r,v}$ be a non-negative continuous variable with a domain $[0, 1]$ to identify requests fully served by vehicle v . Eqs (16)-(18) drives $YR_{r,v}$ to one while $YP_{r,v} = YD_{r,v} = 1$, and drops $YR_{r,v}$ to zero if either of such variables is null.

Eq. (19), causes a vehicle returning to the cross dock from its pick-up trip to perform the unloading operations in just one receiving dock-door ($d \in RD$). Let us define the binary variable $DP_{v,d}$ to denote that the pickup vehicle v has been assigned to the strip dock door d whenever $DP_{v,d} = 1$. In Eq. (19), the set RD includes all the receiving doors available at the receiving dock.

Eq. (20) indicates sequencing pickup vehicles assigned to the same strip dock door. The trucks leave the cross-dock after all freight has been unloaded. Eq. (20) defines a lower bound for the release time (RT_v^P) at which the pickup vehicle v completes the off-load operations at the cross-dock and is ready to perform delivery tasks. We need to this bound to set the value of (RT_v^P) for the vehicle first served at

any receiving dock door. In turn, constraints (21) and (22) relate the times at which vehicles $(v, \hat{v}) \in V$ ($v < \hat{v}$) end their unloading tasks just in case both vehicles have been assigned to the same strip door d ($DP_{v,d} = DP_{\hat{v},d} = 1$). The relative order of a pair of vehicles v and \hat{v} on the queue of the commonly assigned door d is defined by a single variable $ZP_{v,\hat{v}}$ (with $v < \hat{v}$). If $ZP_{v,\hat{v}} = 1$, provided that vehicle v is served before. Otherwise, $ZP_{v,\hat{v}} = 0$ and truck \hat{v} is unloaded earlier. After being serviced at different strip dock doors, the constraints (21) and (22) become redundant. The service time is the sum of two components at every door including a fixed preparation time (ft_w^P) and a variable service-time contribution which directly increases with the cargo to be unloaded given by $\sum_{r \in R} q_r (Y_{P_{r,v}} - Y_{R_{r,v}})$.

Eqs (23)-(24) are representatives for sequencing unloads events at the cross-docks. An unload event n occurs at the cross-dock whenever a pickup vehicle v just completes the discharge of the cargoes to be delivered by other vehicles. Therefore, there will be as many unloads events in the set N as the number of pickup vehicles on duty. N is an ordered event set with the element n occurring before the event \hat{n} ($n < \hat{n}$). Let us define the binary variable $WP_{n,v}$ allocating pickup vehicles to unloads events, and the continuous variable TE_n representing the time at which the event n occurs. The event-time TE_n will be set by the release time of vehicle v from its pickup assignments (RT_v^P) only if $WP_{n,v} = 1$. Eqs (23)-(24) force an inbound vehicle to be exactly assigned to a single time event and an inbound vehicle to be allocated to only one event. Dummy events are those assigned to unused vehicles that will never occur.

Furthermore, Eq. (25) proves the occurrence of event n prior to the event ($n < \hat{n}$). Through Eq (25), the pickup vehicles should be assigned to unloads events in the same order that they complete their pickup duties. If the event n has been allocated to vehicle v ($WP_{n,v} = 1$), then $TE_n = RT_v^P$. Eq (26) sets the value of RT_v^P as a lower bound for TE_n whenever vehicle v has been assigned to either an earlier event ($\hat{n} < n$) or to event n itself. The equality condition is met by Eqs (27)-(30).

The subset of requests already unloaded at the cross-dock at the event time TE_n is involved in Eqs (31)-(33). Let $UR_{r,n,v}$ be a continuous variable with domain $(0,1)$ denoting that request r collected by vehicle v is available for delivery on the cross-dock at the event time TE_n only if $UR_{r,n,v} = 1$. When the request r is not collected by vehicle v ($Y_{P_{r,v}} = 0$) or is assigned to an event $n \neq \hat{n}$ ($WP_{n,v} = 0$), Eqs (31) and (32) drive $UR_{r,n,v}$ to zero. If the reverse situation holds, $UR_{r,n,v}$ is set equal to one by Eq (33).

Continuous variable $UT_{r,n}$ with domain $(0,1)$ provides the subset of requests already unloaded on the receiving dock at time TE_n . If $UT_{r,n} = 1$, then the request r has been discharged from the pickup vehicle at a time earlier than or equal to TE_n . In case the request r still remains on the cross-dock at TE_n , it will be available for delivery at that time. The value of $UT_{r,n}$ is defined by Eq (34).

There are normally some loads temporarily stored in front of the stack doors waiting for the arrival of the other goods to be also delivered by the assigned outbound truck. Eqs (35)-(36) address the further queuing constraints for vehicles assigned to the same receiving door. When the inbound vehicles v and \hat{v} (with $v < \hat{v}$) have been allocated to the same receiving door $d \in RD$ and vehicle v features an earlier unload event ($WP_{n,v} = WP_{\hat{n},\hat{v}} = 1$ with $n < \hat{n}$), then by Eqs. (35) and (36) vehicle v must be served before \hat{v} and $ZP_{v,\hat{v}} = 1$. Otherwise, vehicle v is unloaded

before and $ZP_{v,\hat{v}} = 0$ by Eq. (35). When vehicles v and \hat{v} fail to share the same strip dock door, Eqs (35)-(36) become redundant.

Eq. (37) allocates unloading vehicle to docks. Each vehicle should be allocated to a dock. If the allocation variable $GD_{w,v}$ is equal to 1, vehicle v serves dock w . As stated by Eq. (38), each transportation request must be allocated to a single outbound vehicle. A binary variable $YD_{r,v}$ should be defined to denote the allocation of request r to the outbound vehicle v only if $YD_{r,v} = 1$.

Eq. (38) allocates delivery vehicles to shipping dock doors. We set $DD_{v,\hat{d}}$ as a binary variable allocating outbound vehicles to shipping doors. If $DD_{v,\hat{d}} = 1$, then the loading operations for vehicle v will take place at the shipping door $\hat{d} \in SD$. As stated by Eq. (39), an outbound vehicle on duty must be loaded at just one stack dock door. The set SD comprises the shipping doors available at the cross-dock.

Eqs (40)-(43) aim at identifying the strip and stack dock doors assigned to each vehicle. The continuous variable $DRS_{v,d,\hat{d}}$ with domain $(0, 1)$ has been introduced to indicate that vehicle v should move from the strip door $d \in RD$ to the stack door $\hat{d} \in SD$ before starting the loading operations. Eqs (40)-(43) drive the variable $DRS_{v,d,\hat{d}}$ to one whenever $DP_{v,d} = DD_{v,\hat{d}} = 1$, and drops $DRS_{v,d,\hat{d}}$ to zero if either of such variables are null.

Eqs (44)-(46) clarify the sequence of outbound vehicles assigned to the same shipping door. The continuous variable ST_v^D denotes the time at which the delivery vehicle v starts the loading of the assigned requests at the cross-dock. Considering the same fleet of vehicles for pickup and delivery tasks, a pair of constraints are essential to be defined on the value of ST_v^D :

(a) Pickup assignments need to be completed to let the loading of a delivery vehicle v start, i.e. it shouldn't be earlier than RT_v^P ; and (b) all the preceding trucks on the queue of the assigned stack dock door $\hat{d} \in SD$ (i.e. $DD_{v,\hat{d}} = 1$) should be served to let the loading of vehicle v begin. Eq. (44) accounts for constraint (a) while Eqs (45)-(46) mathematically describe the condition (b) by relating the times ST_v^D and $ST_{\hat{v}}^D$ at which the pair of vehicles $(v, \hat{v}) \in V$ (with $v < \hat{v}$) assigned to the same shipping door \hat{d} ($DD_{v,\hat{d}} = DD_{\hat{v},\hat{d}} = 1$) finish their loading activities at the cross-dock. If vehicle v precedes \hat{v} on the queue of door \hat{d} , then the sequencing variable $ZD_{v,\hat{v}}$ will be equal to one as explained in Eq. (45) applies. Otherwise, $ZD_{v,\hat{v}} = 0$ and Eq. (46) becomes the relevant constraint. When two vehicles are allocated to different stack dock doors, constraints (45)-(46) both become redundant. The total loading time is equal to the sum of a fixed preparation time ft_w^D plus a variable time contribution that directly increases with the load size regarding Eqs (44) and (45)-(46). Furthermore, $tt_{d,\hat{d}}$ states the time spent by a vehicle to move from the receiving door $d \in RD$ to the shipping door $\hat{d} \in SD$. Constraint (44) should be omitted when the fleets of inbound and outbound vehicles are different. If the vehicles are either inbound or outbound trucks, the model will still be applied.

(b) Since there is small travel time between the docks in comparison with the time during which the freights should temporarily remain on the cross-dock, the constraint (44) will keep redundant.

Eqs (47)-(50) assign delivery vehicles to unloads events. All of the requests should be delivered by an available truck at the cross-dock to let an outbound vehicle start its loading. The main reason is that the loading sequence is generally determined by: (a) the need of having the loads tightly packed into the truck and putting the fragile goods on the top, and (b) the ordering of the delivery nodes on the vehicle route [1]. The binary variable $WD_{n,v}$ is defined to denote that the outbound vehicle v has been assigned to the unload event $n \in ND$ only if $WD_{n,v} = 1$. Allocating the outbound vehicle v to event n ($WD_{n,v} = 1$) states that the requests assigned to vehicle v ($YD_{r,v} = 1$) have already been unloaded on the cross-dock at a time earlier than or equal to TE_n . Such requests all feature $UT_{r,n} = 1$ and, therefore, the condition $WD_{n,v} + YD_{r,v} = 2$ implies that $UT_{r,n} = 1$ and the loading of vehicle v is forced to begin after TE_n .

Eq. (47) insures the assignment of each outbound vehicle on duty to a single unload event $n \in N$. Several delivery vehicles can be allocated to the same unload event, though. Eq. (48) will be able to set $WD_{n,v} = YD_{r,v} = 1$ only if the variable $UT_{r,n}$ is equal to one. In this way, Eq. (48) avoids the allocation of event n to an outbound vehicle v if $UT_{r,n} = 0$ for some requests r with $YD_{r,v} = 1$.

Furthermore, Eq. (49) doesn't let an outbound vehicle v allocated to event n to start the loading operations before time TE_n . In addition, Eq. (49) will drive the variable $WD_{n,v}$ to zero if the unload event for vehicle v occurs at some later event ($\acute{n} > n$), i.e. $WP_{\acute{n},v} = 0$ for some ($\acute{n} \leq n$). If every truck is either inbound or outbound, we should omit Eq. (50). We apply similar constraint sets with mathematical structures to the proposed ones for the pickup phase for delivery routes. Replacing the assignment variable $YP_{r,v}$ by $YD_{r,v}$, the routing cost CP_r by CD_r , the visiting time TP_r by TD_r , the sequencing variable $XP_{r,f}$ by $XP_{r,f}$ ($r < \acute{r}$), and the superscript P by D , formulations can be derived from Eqs (6)-(15).

Eqs (51)-(54) define sequencing constraints providing the outbound routing costs from the cross-docks up to the delivery site of request r are defined by Eqs (51)-(54). The parameter M_C^D is a relatively large number. Eqs (55)-(58) elaborate on the set of constraints providing lower bounds for the vehicle stop times at delivery locations. Eq. (59) forces the load transported by vehicle v not to exceed its maximum capacity (Q_v). Eq. (60) states that the load transported by cross-dock w cannot exceed its maximum capacity (Q_w).

Eqs (61)-(62) demonstrate the requirement of further queuing constraints for vehicles sharing the same shipping door. If delivery vehicles v and \acute{v} are loaded at the same stack dock door and vehicle v is allocated to an earlier event, then vehicle v will be served before and $ZD_{v,\acute{v}} = 1$ regarding Eqs. (61)-(62). In the contrary, vehicle v' is loaded earlier and $ZD_{v,\acute{v}} = 0$. The value of $ZD_{v,\acute{v}}$ can also be meaningless when the vehicles have been allocated to different shipping doors.

Related constraints to the total routing cost and the vehicle arrival times are considered as additional constraints to speed up the solution process. If there is a relationship between arrival time AT_v^P and the total routing cost for the pickup tour of vehicle v , lower and upper bounds on the value of AT_v^P will be obtained through Eqs (63) and (64), respectively. Estimating AT_v^P as the sum of the starting time ST_v^P plus the total service time at the visited locations and the total traveling time, such bounds will be obtained. Regardless of the time windows for the service start at the P/D locations, the parameter η will be equal to zero. Nonetheless, it has been chosen

$\eta_p = 0 \cdot 001$ to account for round off errors. Since the pickup vehicles sometimes are required to wait for the opening of the time window at some visiting sites, the value of η_p should be increased to 0.1-0.3 for problems with narrow time windows.

Constraints (65)-(66) that are similar to Eqs (63)-(64) are dedicated to the delivery phase. Eq. (67) explains the valid inequality constraints for allocating received dock doors to the vehicles. Constraint (67) is incorporated into the mathematical model to solve large problems and eliminate symmetric solutions. If the set RD comprises three elements $\{rd_1, rd_2, rd_3\}$, then constraints (67) allocates the dock door rd_1 to the vehicle v^* that first unloads the cargo on the cross-dock terminal (e.g., $WP_{n1,v^*} = 1$), the dock door rd_2 to the vehicle $v^\#$ completing the unloading operations in the second place (e.g., $WP_{n2,v^\#} = 1$) and rd_3 to the truck finishing the pickup duties on third place. The optimal solution is not excluded from the feasible region by Constraint (67) but avoids symmetrical assignments.

4. Solution methodology

4.1. Fuzzy-interactive approach

Mathematical programming techniques and equations vividly fail to solve transportation decision-making problems by fuzzy objective function coefficients. Accordingly, a fuzzy-interactive multi-objective linear programming model is presented to overcome this deficiency while facing transportation decision problems with fuzzy objective function coefficients. The proposed model has proved to be far better than the previous ones in terms of computational flexibility and efficiency. As it is obvious from the presented model, the majority of parameters are fuzzy and include correct values and technological coefficients. Moreover, objective functions and main deterministic variables are set as the constraints.

4.1.1. Deterministic equivalent model

Two-phase approach in order to solve the proposed fuzzy model:

First phase, the initial fuzzy model is altered to a deterministic equivalent auxiliary model.

The second phase, a fuzzy method is applied to obtain the final preferred compromise solution.

4.1.2. Fuzzy solution approach

Computational Results

This section is trying to analyze the results of solving the proposed model. Due to test the accuracy of the proposed model, several problems are solved in GAMS. This model mainly aims at minimizing costs. Models outputs and objective functions value are presented in the last part of this section. Regarding this fact that the majority of model parameters are fuzzy and it also includes parameters of right values and technological coefficients and considers objective functions and main deterministic problem variables as constraints, a two-phase approach is applied to solve the proposed fuzzy model. We transform the initial fuzzy model into a deterministic

equivalent auxiliary model in the first phase. Then we apply a fuzzy method to obtain the final preferred compromise solution, in the second step.

In order to generate triangular fuzzy numbers, we also estimate three sensitive points (the most probable value, the pessimistic value, and the optimistic value). To achieve this purpose, we generated the most probable value (c^m) of every parameter at first. In the next step, we used normal distribution to generate two random numbers (r_1, r_2) between 0.2 and 0.8 without changing the generality of the problem. Then, the pessimistic (c^p) and optimistic (c^o) values were calculated using the fuzzy numbers [23]. Being solved in GAMS, The model resulted in answers to the objective functions that are analyzed in the fuzzy mode.

Considering numerical solutions different dimensions, it included 4 cross docks, 3 doors (entrance and exit), 8 types of vehicles, and 40 types of customer demands. The problem of different dimensions solved in GAMS. The input values of parameters helped us in solving these problems.

In this section number of tables is considered, in which the values of the parameters are inserted. Considering the variety of case studies we have examined, our reviews show that these values are applicable in the real dimension.

Table (2). The speed of a vehicle (v)

v	1	2	3	4	5	6	7	8
sp _v	70	70	70	70	70	70	70	70

Table (3). Vehicle transportation time from a receiving door ($d \in RD$) to a sending door ($\acute{d} \in SD$)

tt _{d,đ}	RD1	RD2	RD3
RD1	2	4	8
RD2	4	2	5
RD3	7	6	2

Table (4). Vehicles cost per a distance unit (v)

v	1	2	3	4	5	6	7	8
uc _v	200	230	210	270	200	230	210	270

There are many differences between the fuzzy mode and the definite mode in terms of objective function values. This difference proves the importance of considering uncertainty. Furthermore, the problem is not feasible for certain values of α . This problem solution is offered in Table (5).

Table (5). Objective Functions solutions for Different Parameters of α in the Fuzzy Mode

α	0.4	0.5	0.6	0.7	0.8	0.9	0.95	1
z_1	8406835.676	7354925.293	6371896.774	5458032.107	4613174.659	3837059.586	3474780.440	3129893.391
z_2	1215756.610	1063283.697	920818.388	788411.837	665978.677	553518.909	501029.047	451032.532
z_3	8409594.156	7357338.019	6373986.425	5459821.455	4614686.417	3838316.386	3475918.247	3130917.928

These tables prove our claim about the significant difference between the definite and fuzzy modes, and consequently, the importance of considering uncertainty in the model. Due to providing solutions for the fuzzy multi-objective model, we applied the proposed approach. We consider different values of θ were considered; nonetheless, a preferable value is selected by the decision-maker. In order to obtain the functions weights in the presence of different objective functions, we can apply other decision-making methods such as AHP. Tables (6) and (7) represent the

sensitivity analysis (SA) performed on the problem parameters within different positions. In other words, the values of membership functions are presented in Table (6) for different parameters of α , and the final results of solving the model for α , β , and γ are presented in Table (7):

Table (6). The Values of the Membership Functions for Every Objective Function for Different Values of α

α	μ_1	μ_2	μ_3
0.8	0.601384482	0.714405423	0.600220716
0.9	0.230709369	0.252944375	0.229718001
0.95	0.155614454	0.124077384	0.154744266
1	0.081764138	0.028796304	0.080958103

Table (7). The Final Results of Solving the Model for α , β , and γ

α	γ	θ	Z_1	Z_2	Z_3	λ	α	γ	θ	Z_1	Z_2	Z_3	λ
0.8	0.9	(0.2·0.35·0.45)				0.5732	0.9	0.5	(0.2·0.35·0.45)				0.1780
		(0.8·0.15·0.05)				0.5972			(0.8·0.15·0.05)				0.2239
		(0.5·0.23·0.27)	4988200	719680	4989100	0.5840			(0.5·0.23·0.27)	4848400	697160	4848700	0.1987
		(0.37·0.33·0.3)				0.5822			(0.37·0.33·0.3)				0.1952
		(0.02·0.58·0.4)				0.5762			(0.02·0.58·0.4)				-

These tables evidently prove that objective functions are highly sensitive to changes in γ for different parameters of α . When $\alpha= 0.9$, the sensitivity is higher. In other cases, changes of γ are relatively less effective. In order to make decisions for selecting a position, the decision-maker considers all of certain conditions.

Conclusion and Suggestions for Future research

Supply chain management has convinced many researchers within recent years aiming at facilitating industrial companies and organizations, particularly in developed countries. A fruitful approach through supply chain management is the implementation of lean production and consequently, a lean supply chain. On the other hand, a successful supply chain can't be considered without designing appropriate cross docks; accordingly, logistics companies warmly welcome cross-docks in large-scale transportations. Cross docks set their target as applying the main policy for aggregating products within warehouses. Instead of being sent to customers, required demands from different suppliers will be aggregated in cross docks. In order to reduce the transportation costs, the products are better to be classified regarding customer demands and then, sent to destinations.

With respect to Dando and Cerda's model [21] on scheduling and planning cross-docking operations, we extended our model by adding several cross docks in this paper. To consider the uncertainty of fuzzy parameters, an interactive fuzzy approach was offered. There is no way to encounter the inaccurate and uncertain nature of parameters and model them except using a fuzzy distribution. Mathematical programming techniques vividly fail to solve transportation decision-making problems by fuzzy objective function coefficients. To overcome this deficiency, we provided a fuzzy-interactive multi-objective linear programming model for solving transportation decision problems by fuzzy objective function coefficients in this paper. The proposed method proved to be flexible and efficient, computationally.

As mentioned earlier, our proposed approach is a two-phase method that converts the initial fuzzy model to an equivalent auxiliary definite one in the first phase, and applies a fuzzy method to obtain the approved preferable solution, in the second step. Using both definite and fuzzy methods for different parameters, a couple of numerical examples were conducted. Then, the final solutions were compared and analyzed. The results show a high level of similarity between definite and fuzzy solutions while using certain parameters and a high level of difference while using uncertain parameters to prove the importance of considering uncertainty.

No wonder, this problem is categorized as an NP-hard one regarding its time consumption and computational complexities. Due to the extensive nature of cross-dock problems, different methods have been offered to solve them. In order to change the previous models to more flexible versions, new assumptions can also be considered. Moreover, metaheuristic algorithms can also be suggested to interested readers.

References

- [1] Van Belle, J., Valckenaers, P., Cattrysse, D. (2012) "Cross-docking: State of the art", *Omega*.
- [2] Boysen, N., Fliedner, M., Scholl, A. (2010) "Scheduling inbound and outbound trucks at cross-docking terminals", *OR spectrum*, 32, pp. 135-161.
- [3] Yu, W., Egbelu, P. J. (2008) "Scheduling of inbound and outbound trucks in cross-docking systems with temporary storage", *European Journal of Operational Research*, 184.
- [4] Li, Y., Lim, A., Rodrigues, B. (2004) "Cross-docking-JIT scheduling with time windows", *Journal of Operational Research Society*, 55, pp. 1342-1351.
- [5] Wisittipanich, W., Hengmeechai, P. (2017) "Truck scheduling in multi-door cross-docking terminal by modified particle swarm optimization", *Journal of Computers & Industrial Engineering*, 113, pp. 793-802.
- [6] Serrano, C., Delorme, X., Dolgui, A. (2017) "Scheduling of truck arrivals, truck departures and shop-floor operation in a cross-dock platform, based on trucks loading plans", *International Journal of Production Economics*, 194, pp. 102-112.
- [7] Molavi, D., Shahmardan, A., Sajadieh, M. S. (2018) "Truck scheduling in a cross-docking systems with fixed due dates and shipment sorting" *Journal of Computers & Industrial Engineering*, 117, pp. 29-40.
- [8] Amini, A., Tavakkoli-Moghaddam, R. (2016) "A bi-objective truck scheduling problem in a cross-docking center with the probability of breakdown for trucks", *Journal of Computers & Industrial Engineering*, 96, pp. 180-191.
- [9] Keshtzari, M., Naderi, B., Mehdizadeh, E. (2016) "An improved mathematical model and a hybrid metaheuristic for truck scheduling in cross-dock problems", *Journal of Computers & Industrial Engineering*, 91, pp. 197-204.
- [10] Ahkamiraad, A., Wang, Y. (2018) "Capacitated and multiple cross-docked vehicle routing problems with pickup, delivery, and time windows", *Journal of Computers & Industrial Engineering*, 119, pp. 76-84.
- [11] Bazgosha, A., Ranjbar, M., Jamili, N. (2017) "Scheduling of loading and unloading operations in a multi stations transshipment terminal with release date and inventory constraints", *Journal of Computers & Industrial Engineering*, 106, pp. 20-31.

- [12] Hasani Goodarzi, A., Zegordi, S. H. (2016) "A location-routing problem for cross-docking networks: A biogeography-based optimization algorithm", *Journal of Computers & Industrial Engineering*, 102, pp. 132-146.
- [13] Assadi, M. T., Bagheri, M. (2016) "Differential evolution and Population-based simulated annealing for truck scheduling problem in multiple door cross-docking systems", *Journal of Computers & Industrial Engineering*, 96, pp. 149-161.
- [14] Maknoon, M.Y., Kone, O., Baptiste, P. (2014) "A sequential priority-based heuristic for scheduling material handling in a satellite cross-dock", *Journal of Computers & Industrial Engineering*, 72, pp. 43-49.
- [15] Azimi, P. (2015) "On-line cross-docking: A general new concept at a container port", *Scientia Iranica*, 22, pp. 2585-2594.
- [16] Motaghedi-Larijani, A., Aminnayeri, M. (2018) "Optimizing the number of outbound doors in the cross-dock based on a new queuing system with the assumption of beta arrival time", *Scientia Iranica*, 25, pp. 2282-2296.
- [17] Chen, P., Guo, Y., Lim, A., Rodrigues, B. (2006) "Multiple cross docks with inventory and time windows", *Computers & Operations Research*, 33, pp. 43-63.
- [18] Lee, Y.H., Jung, J.W., Lee, K.M. (2006) "Vehicle routing scheduling for cross-docking in the supply chain", *Computers & Industrial Engineering*, 51, pp. 247-256.
- [19] Wen, M., Larsen, J., Clausen, J., Cordeau, J.F., Laporte, G. (2009) "Vehicle routing with cross-docking", *Journal of the Operational Research Society*, 60, pp. 1708-1718.
- [20] Miao, Z., Lim, A., Ma, H. (2009) "Truck dock assignment problem with operational time constraint within cross docks", *European journal of operational research*, 192, pp. 105-115.
- [21] Dondo, R., Cerdá, J. (2012) "A Sweep-Heuristic based formulation for the vehicle routing problem with cross-docking", *Computers & Chemical Engineering*.
- [22] Mousavi, S. M., Tavakkoli-Moghaddam, R. (2013) "A hybrid simulated annealing algorithm for location and routing scheduling problems with cross-docking in the supply chain", *Journal of Manufacturing Systems*, 33, pp. 335-347.
- [23] Rajabi, M., Akbarpour Shirazi, M. (2016) "Truck scheduling in a cross-dock system with multiple doors and uncertainty in availability of trucks", *Journal of Applied Environmental and Biological Sciences*, 6, pp. 101-109.
- [24] Dubois, D., Fargier, H., Fortemps, P. (2003) "Fuzzy scheduling: Modeling flexible constraints vs. coping with incomplete knowledge", *European Journal of Operational Research*, 147, pp. 231-252.
- [25] Baykasoğlu, A., Göçken, M., Unutmaz, D. (2008) "New approaches to due date assignment in job shops", *European Journal of Operational Research*, 187, pp. 31-45.
- [26] Liang, H., Wang, N. (2005) "Partially linear single-index measurement error models", *Statistica Sinica*, 15, pp. 99-116.