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Work Zone Traffic Simulation using Cellular Automata

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Abstract

The focus of the present work is to develop a simulation model using cellular automata for work zone traffic containing homogeneous traffic condition. Work zone is created by blocking some of the cells. It can be seen that the lane drop not only produces a jam in the blocked lane as well as in the bypass lane. Space-time plots and speed distribution plots are obtained for various cell occupancies and the effect of lane drop is presented. Reduction in capacity due to the lane drop is obtained and the results are compared with the values suggested by Highway Capacity Manual.

Keywords: Cellular automata, Parallel update scheme, Microscopic properties, Macroscopic properties, Lane drop.

1. Introduction

There are circumstances where there is a drop in the lane width due to construction activities. Under such condition, drop in the road capacity occurs. Determination of capacity for such condition is important to manage and plan the traffic flow. First cellular automata model, NaSchmodel, for vehicular traffic simulation was for single lane traffic (Nagel and Schreckenberg, 1992), which was further modified by other researchers. (Takayasu and Takayasu, 1993) modified the basic model by introducing delay in acceleration. (Benjamin et al., 1996)included certain additional rules to the basic NaSch model, which is known as BJH model. (Barlovic et al.,1998) introduced an improved generation of slow-to-start cellular automaton model, which could successfully demonstrate metastable states and hysteresis.(Helbing and Schreckenberg,1999) developed a one dimensional cellular automaton model to describe stop and go conditions. (Knospe et al., 2000) proposed the brake light model, which implements dynamic interaction of vehicles. (Li et al., 2001) proposed another variant of cellular automaton model for one lane traffic flow. (Bham and Benekohal, 2004) developed CELLSIM model to simulate high volume traffic, which uses the concept of cellular automata and car-following models. (Clarridge and Salomaa, 2010) modified BJH model to incorporate slow-to-stop rules to demonstrate deceleration properties well to avoid abrupt and sudden deceleration characteristics when meeting a static or a slow vehicle.

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The occurrence of single lane road is very rare. Major roads in real life scenario comprise of two or more lanes. Cellular automaton models were modified to incorporate longitudinal movement and lateral movement of vehicles by researchers such as (Biham et al.,1992; Nagatani,1993; Rickert et al., (1996); Benjaafar et al., (1997); Knospe et al., (1999); Knospe et al.,(2002); Jia et al.,(2005); Li et al.,(2006); Lan et al.,(2009); Rawat et al.,(2012); Lv et al., (2013); Zhu et al., (2015) and Guo (2016).

Di Gregorio et al., (2008) proposed STRATUNA, a cellular automata-based model used for simulating the evolution of two/three lane highways. The proposed model proves to be useful for traffic forecasting and economical point of view as the model gives feedback to link different highway designs to different congestion toll charges through an established cost system. (Zhu et al. 2009) studied the effect of blockage induced by an accident car using both symmetric and asymmetric lane changing rules. (Srikanth et al. 2017) studied the lane changing behavior with homogeneous vehicle type traffic on highway sections using simulation model. (Meng, 2010) calibrated the randomization probability parameter of the cellular automata model used to simulate work zone traffic. (Arasan and Arkatkar, 2011) studied the relationship between traffic flow variables and quantified the vehicular interactions. (Meng and Weng, 2011) developed cellular automata model to simulate heterogeneous traffic at work zone and determine traffic delay. (Singh et al., 2017) worked on the modeling of work zone traffic flow using cellular automata and studied queue length and delay. Lateral movement rules were modified to give a realistic movement in the work zone. A rational relationship was found between delay and work zone length.

Literature review reveals that feeble amount of work has been done to determine capacity of work zone traffic using cellular automata, which forms the motivation of the present study.

2. Proposed Model

This section presents various assumptions and calculations performed in the model.

a) Discretization of flow space: In order to accomplish the simulation, a road length of one kilometer and different widths (i.e. 7.0m, 10.5m and 14m) is considered. The flow space is discretized into square cells of 0.1m. Closed loop simulation with parallel update scheme is used in the present study.(Pal and Mallikarjuna, 2010) decided the cell size depending on the size of the vehicles, its lateral position, lateral gaps on either side of the vehicle.

For different widths of roads as well as obstructions, the length of obstruction remains the same i.e. running between the lengths 500m (5000th cell) to 650m (6500th cell). Figure 1 shows the schematic diagram of a 7m road with an obstruction of width 3.5m running between 500m to 650m.

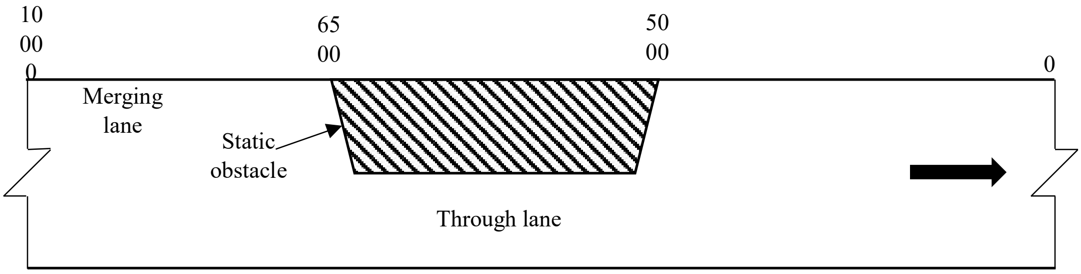

Figure 1: Schematic diagram showing a lane drop for two-lane road (figure not to

b) Perception reaction time and system update time: The perception reaction time and system update time adopted for the present study is 0.5s. With the system update time being 0.5s, a vehicle moving one cell in one time step (i.e. Δ=0.5s) attains a velocity of 0.72km/h. Hence, the model can sense a minimum variation of speed of 0.72km/h.
c) Size of different vehicles assumed in the present study: The simulation is performed for homogeneous traffic condition (i.e. cars only).The size of cars assumed in the present study are same as that used by (Arasan and Koshy,2005) i.e. 4.2m long and 1.7m wide.
d) Calculation of steering angle: A maximum value of steering angle of 20^{o} is adopted for the present study. According to (Lv et al.,2013) a maximum steering angle change of 20^{o} gives a safe and comfortable ride.
e) Desirable speed of drivers: Three types of drivers are assumed in the present study i.e. aggressive drivers, normal drivers and slow moving drivers.
f) Transverse Clearance: The values of lateral clearance are adopted from (Arasan and Koshy, 2005) with extrapolation of speed values.
g) Actual Gap (G_a), Perceived Gap (G_p) and Buffer Space (B):shown in Figure 2.

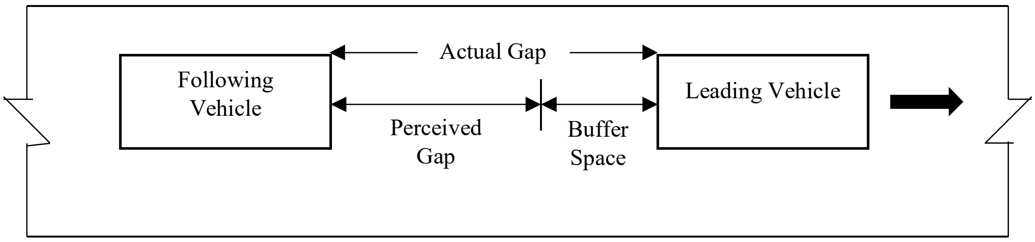

Figure 2: Actual gap, perceived gap and buffer space

i. Actual gap (G_a): In real world scenario, every vehicle is separated from the preceding vehicle by some distance, considered as actual gap (G_a).
ii. Buffer space (B): The amount of space, which a vehicle reserves in front of it to maintain a safe distance from the lead vehicle is termed as buffer space (B) (Chattaraj et al., 2013). It is the psychological spacing between two consecutive vehicles, which can be utilized during emergency condition. Mathematically buffer space can be represented as

$$B = C_1 + C_2 v \tag{1}$$

Where C_1 and C_2 are calibration constants and v is the speed of the considered vehicle as calculated in the previous time step.

iii. Perceived gap (G_p): The amount of space, which is used for movement of vehicles, is known as perceived gap. It is the space which a driver perceives safe for its movement (Chattaraj et al., 2013). Mathematically it is the difference between actual gap and buffer space.

$$G_p = G_a - B \quad (2)$$

The concept of actual gap, buffer space and perceived gap is shown in Figure 1.

h) Relative Speed: The following vehicle when impeded by a lead vehicle always tries to take a better path by changing its trajectory, either to the right or to the left by changing the steering angle. The following vehicle before changing the steering angle checks for the relative speed i.e. speed of LV minus speed of FV. If the relative speed is negative, the FV decides to change the steering angle.

3. Implementation of the Model

In this section steps about the implementation of the proposed model for simulating unidirectional uninterrupted traffic flow are detailed.

Step 1:

This step is used to input the road geometry and generate vehicles using random numbers. As the simulation process is carried out for a closed loop system, the number of vehicles remain same throughout the simulation process. Besides assigning desirable speeds, every vehicle is also assigned an arbitrary speed, which is less than desirable speed. This arbitrary speed is used only in the first time step of the simulation. The remaining simulation process is carried out using the updated speeds of the vehicles.

Step 2:

In this step, the buffer space (B) is computed using Equation 1. Actual gap (G_a) and perceived gap (G_p) are computed using Equation 2. Equation 1 uses the speed values obtained in the previous time step. If the value of speed is higher, value of B becomes higher. It may happen that value of B is higher than that of G_a. Hence, a check is performed to confirm whether the buffer space computed is greater than G_a.

If $B > G_a$

$$v = v.\left({G_a}/{B} \right)^n \quad (3)$$

Else

Calculate G_p as per Equation 2

Where ($n \in Z$, $1 \leq n \leq 100$)

There may be circumstances when value of G_a is large; v is small as a result the buffer space computed (B) becomes small. Under such circumstances, value of G_p becomes large. Hence a vehicle may move from very low speed to very high speed in one time step (i.e. Δ=0.5s) which is practically impossible. Hence, a limiting value of acceleration of 1.4m/s^2 (AASHTO 2001) is adopted for the simulation. Acceleration is calculated using the formula

$$v^2 - u^2 = 2aS \quad (4)$$

Where, *v* is the speed of the vehicle in the present time step, u is the speed of the vehicle in the previous time step, a is the acceleration and S is the distance covered.

It may be noted that Equation 2 takes care about the acceleration as well as deceleration of vehicles. Sometimes it may happen that a vehicle undergoes sudden deceleration, which is beyond comfortable deceleration ($3.4m/s^2$, AASHTO 2004). Such actions are to avoid collision with the vehicle ahead. However, the occurrence of such situation is very rare as B is adjusted according to the value of G_a as per Equation 3 at every instant of time.

A vehicle decides to move forward using the equations 1,2,3 and 4. Using Equation 1, the buffer space (B) is calculated. Using the value of buffer space, the perceived gap (G_p) is computed. If due to higher values of v, the computed B is greater than actual gap (G_a), then the speed of the vehicle is reduced using Equation 3. At every time step, the acceleration and deceleration (calculated using Equation 4) of the vehicle is checked in order to confirm that they remain within permissible limit as suggested by AASTHO. This process is repeated for each vehicle in every time step.

Step 3:

In this step, decision for changing the steering angle is taken. This step is to check the availability of lateral space at the vicinity of the test vehicle either to the left or to the right (see Figure 3). The length of the vacant lateral space extends up to half of the length of the vehicle (at the rear side) under consideration. The decision for moving the steering in the right or left direction is taken by the test vehicle depending on the position of its pivot with respect to the pivot point of the leading vehicle. The sequence of decision-making process is shown in Figure 4.

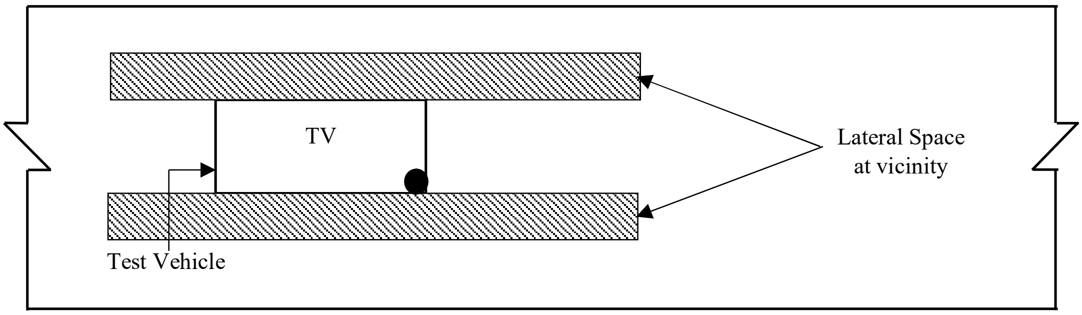

Figure 3: Check of lateral vacancy space used in the proposed model

In Figure 4 (a) (Situation-1) the pivot point of the following vehicle (FV) lies to the right hand side of the symmetric line (LL´) dividing the leading vehicle (LV) into two longitudinal halves.
Figure 4 (b) (Situation-2) may be considered as a special case of Situation-1 where the pivot point of the LV and FV lie in the same line.
Figure 4 (c) (Situation-3) is as similar to Situation-1 where the pivot point of the FV lies to the left of the line of symmetry LL´.

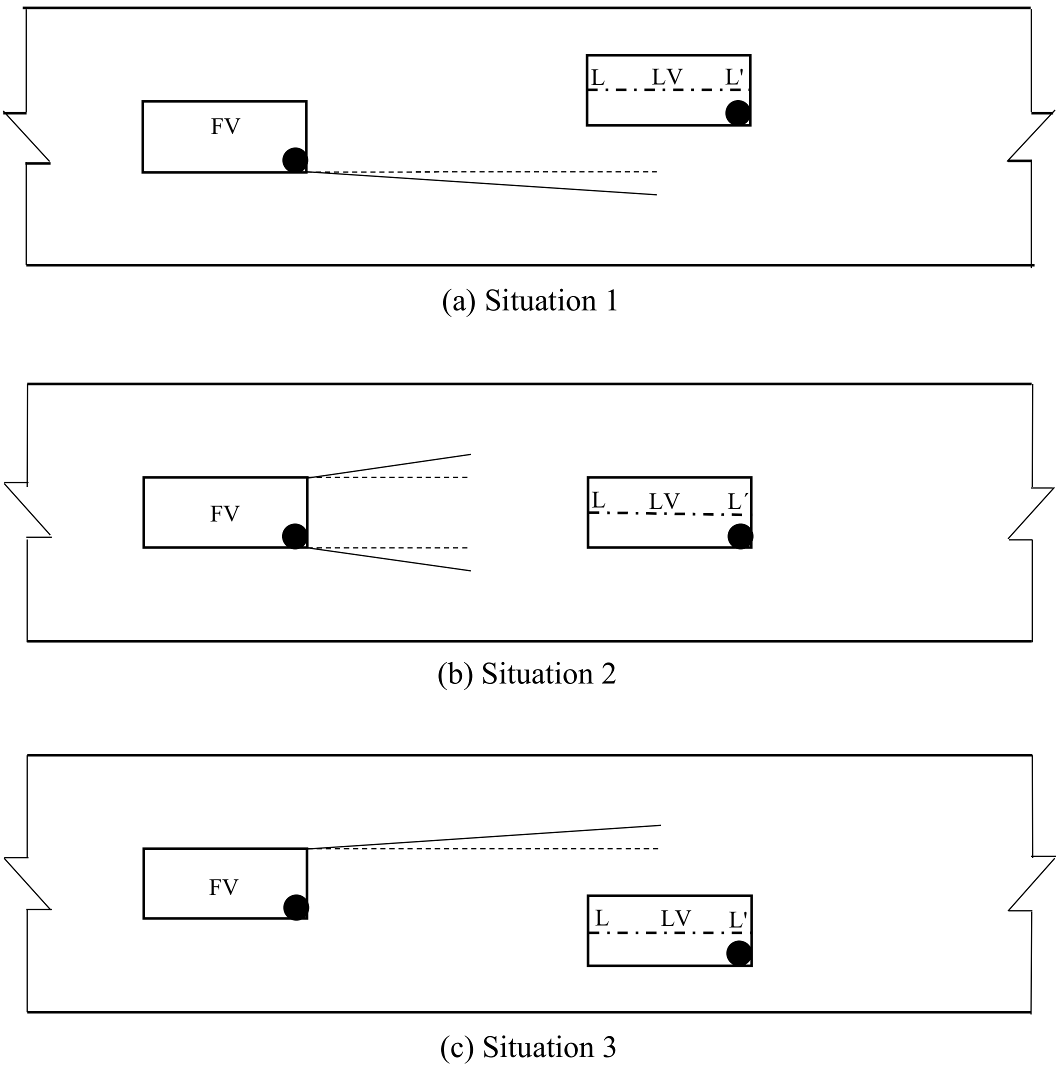

Figure 4: (a) Criteria for a right steering movement (b) Criteria for either a right or a left steering movement (c) Criteria for a left steering movement

Step 4:

In this step the new position of the vehicle is updated (in terms of x and y coordinates) and the flow parameters like speed, flow and density are updated.

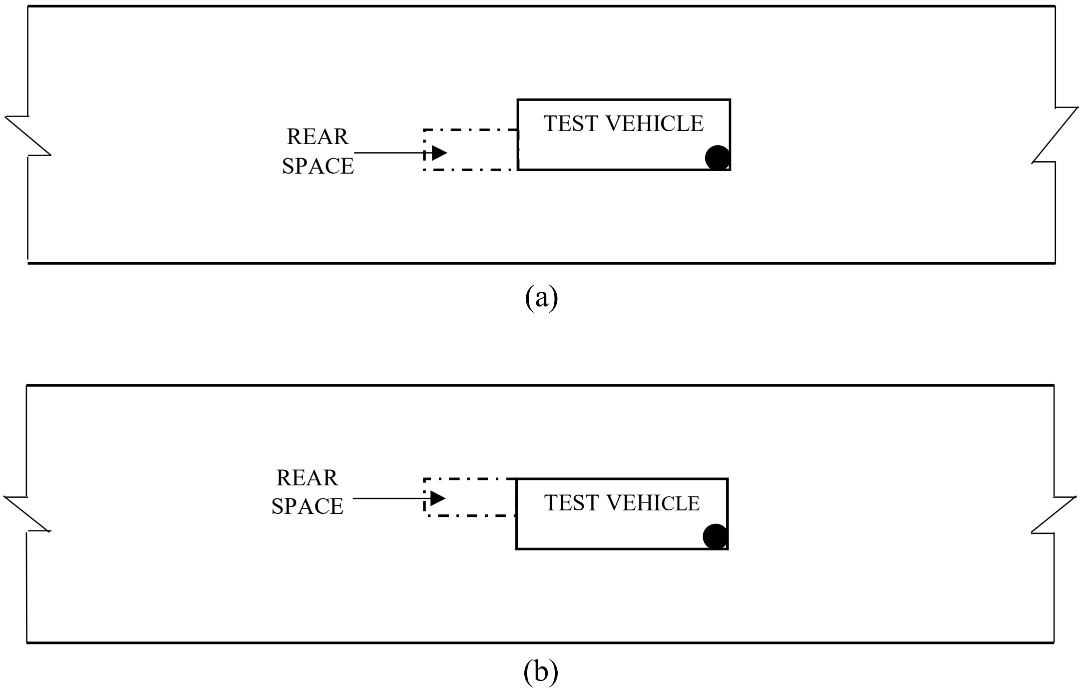

Figure 5: (a) Rear space check for a vehicle intending a right steering movement
(b) Rear space check for a vehicle intending a left steering movement

4. Results

Various microscopic and macroscopic properties explained by the model are illustrated in the present section.Figure 6, Figure7 and Figure8 explain the important microscopic properties, namely, Local Stability, Asymptotic Stability andClosing-in/Shying-away behavior, respectively.

4.1 Study of Local stability

When any kind of disturbance is introduced to the leading vehicle (LV) (either by acceleration or by deceleration) for certain time steps, the following vehicle (FV) adjusts itself such that the system of LV-FV stabilizes after some time. The disturbances introduced into the system dampens gradually and die out after some time. This phenomenon of damping and dying out of perturbations is termed as local stability.

Figure 6 gives the speed versus time and distance headway versus time plots for local stability. It can be seen that the system of LV and FV attain a constant distance headway after certain time which shows that the proposed model is locally stable. The constant distance headway attained is termed as safe distance headway.

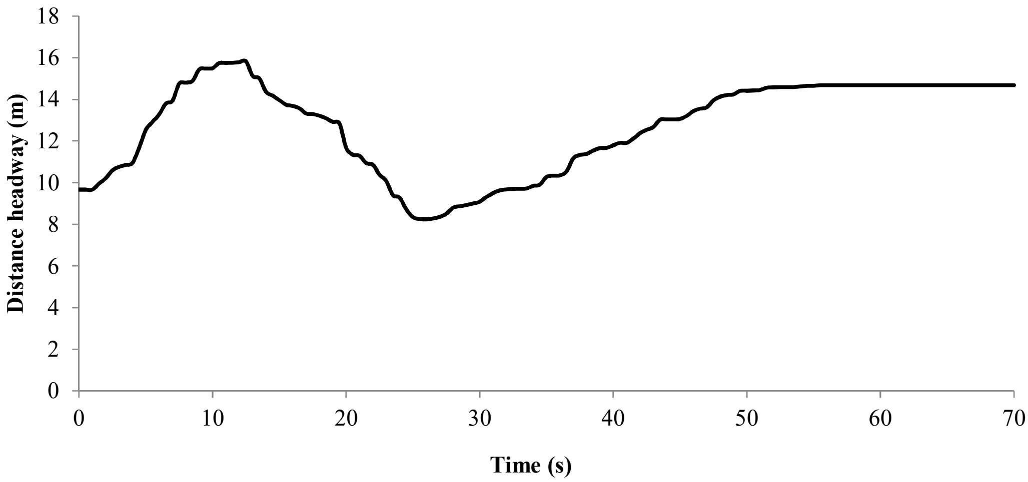

Figure 6: Local stability using proposed model

4.2 Study of asymptotic stability

In a platoon of vehicles, the disturbance created by the leading vehicle gradually dampens while moving upstream. For example, in a platoon of five vehicles the extent of disturbance felt by the fifth and fourth vehicle is less than the disturbance felt by the third and second vehicle. This damping of perturbation is known as asymptotic stability.

Figure 7 shows asymptotic stability. A platoon of four vehicles are studied. In the Figure 7, dh1-2 refers to distance headway between first and second vehicle and so on. It can be seen from Figure 7 that the perturbation gets damped as the chronology of vehicle increased towards upstream. This confirms the asymptotic stability of the system of vehicles.

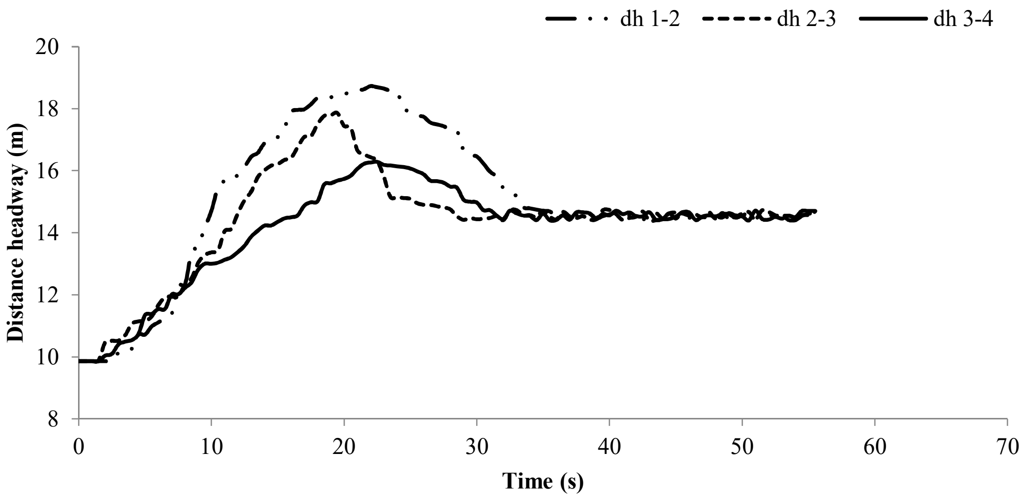

Figure 7: Asymptotic stability using proposed model

4.3 Study of closing-in and shying-away behavior

In a system of LV and FV when the FV is far away from the LV, the FV accelerates so as to reduce the distance between itself and the LV so that the inter-vehicular distance reduces to a safe distance. This distance can also be referred to as safe distance headway (SDH). This behavior of FV is known as closing-in behavior. Similarly, if the FV is very close to the LV, then the FV reduces its speed until the distance headway between the LV and FV becomes equal to SDH. This behavior is referred to as shying-away behavior.

The inter-vehicular distance is kept higher than safe distance headway (SDH) and the ISLV is equal to ISFV for the study of closing-in behavior. No perturbation is introduced to the system of LV and FV. The dotted line in Figure 8 indicates the closing-in behavior. As mentioned earlier, the FV accelerates to move closer to the LV. The distance headway becomes stable at 10.24m.

The inter-vehicular distance is kept lower than SDH and ISLV is equal to ISFV. There are no perturbations to the system of LV and FV. The solid line in Figure 8 indicates shying-away behavior. The distance headway becomes stable at 10.24m.

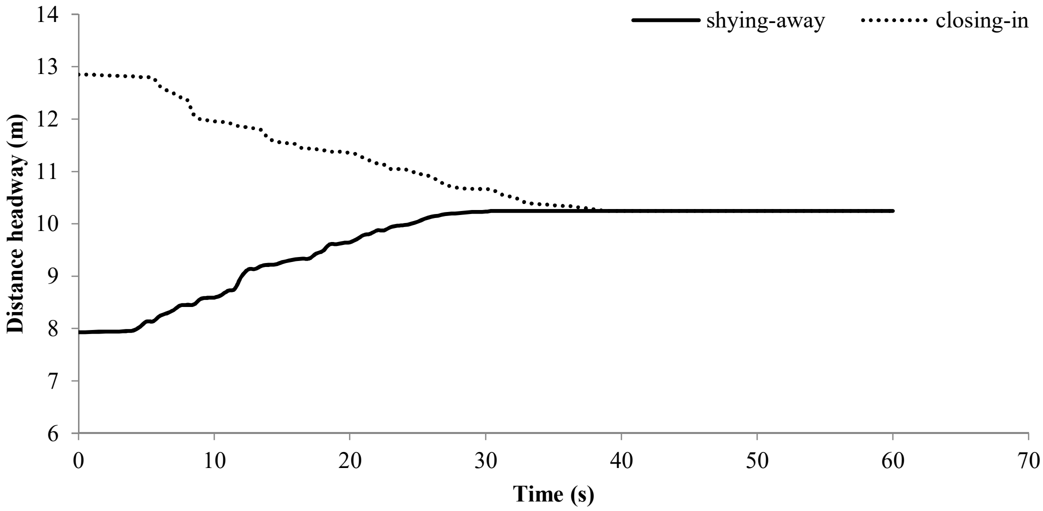

Figure 8: Closing-in and shying-away behavior using proposed model

There are circumstances where there is a drop in the lane width due to construction activities which affect the macroscopic properties. Under such condition, drop in the road capacity occurs. Determination of capacity for such condition is important to manage and plan the traffic flow. Figure 9 explains the Space-time plot for a two-lane road with one lane blocked for two different cases. Figure 10 explains the Speed distribution for a two-lane road with one-lane blocked (both at upstream and downstream).

4.4 Space-time plot

Road width of 7m and length one kilometer (Case-1) is considered for the simulation. An obstruction of width 3.5m and running between the lengths 500m to 650m (i.e. 5000th cell to 6500th cell) is introduced into the road width (Refer Figure 1). Despite the presence of static obstacle, the vehicles can move freely without any much hindrance at lower cell occupancy (density). However, with increase in density, the vehicles form cluster at the upstream of the obstacle. Certain vehicles wait for the congestion to disperse, before moving ahead. On further increasing the density, there is a formation of a long queue at the upstream of the obstacle, which propagates until the upstream of the road section. As this is a closed-loop simulation, formation of jam at the upstream side would lead to the formation of jam at the downstream also. This happens, because the vehicles from the downstream are not capable of relocating at the upstream of the flow space. This phenomenon is more pronounced at higher densities. In order to represent the above phenomenon, space-time plots for two different cell occupancies i.e. 5% and 15% are presented. These two cell occupancies are chosen to represent the space-time plots at low and high densities respectively. Space-time plot for a two-lane road with one lane blocked condition is shown in Figure 9.

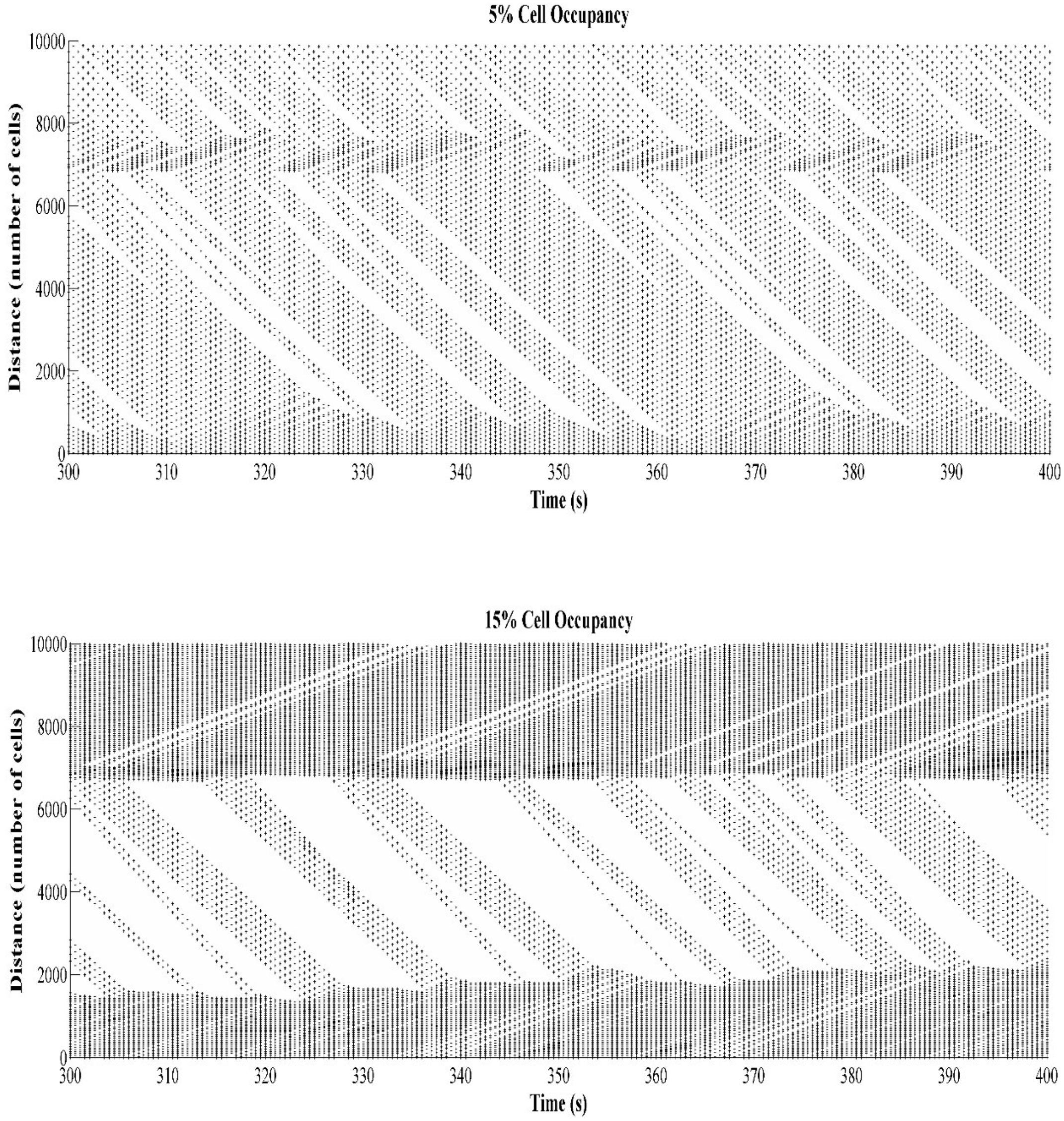

Figure 9: (a) Space-time plot for a two-lane road with one lane blocked at 5% cell occupancy

(b) Space-time plot for a two-lane road with one lane blocked at 15% cell occupancy

As seen from the Figure 15, the vehicles move uninterrupted at a cell occupancy of 5%. There is no congestion at the upstream or the downstream. However as the occupancy is increased to 15%, jams can be seen both at the upstream and the downstream side. It can be assumed that the speed of the vehicles remains almost unaffected both at upstream and downstream side for cell occupancy of 5% however at a cell occupancy of 15% the speed values must drop significantly both for the upstream and downstream section.

4.5 Speed distribution

Figure 10 presents the speed distribution of a two-lane road at the upstream and downstream side at 5% and 15% cell occupancy. Comparing Figure 10 (a) and (b) it can be seen that there is a drop in speed values at the upstream side. This is attributed to the presence of congestion at the upstream side nearer to the obstruction whereas due to low density, the number of vehicles at the downstream side remains less. Hence, vehicles at downstream side attain higher speed. However, when we compare Figure 10 (c) and (d) there is minimal variation in speed values as the congestion occurs at the upstream as well as downstream side (see Figure 9).

Comparing Figure 10 (a) (c) pair and (b) (d) pair there is a remarkable variation in speed values as the cell occupancy increases. This happens due to the congestion at the upstream and downstream side at higher cell occupancy.

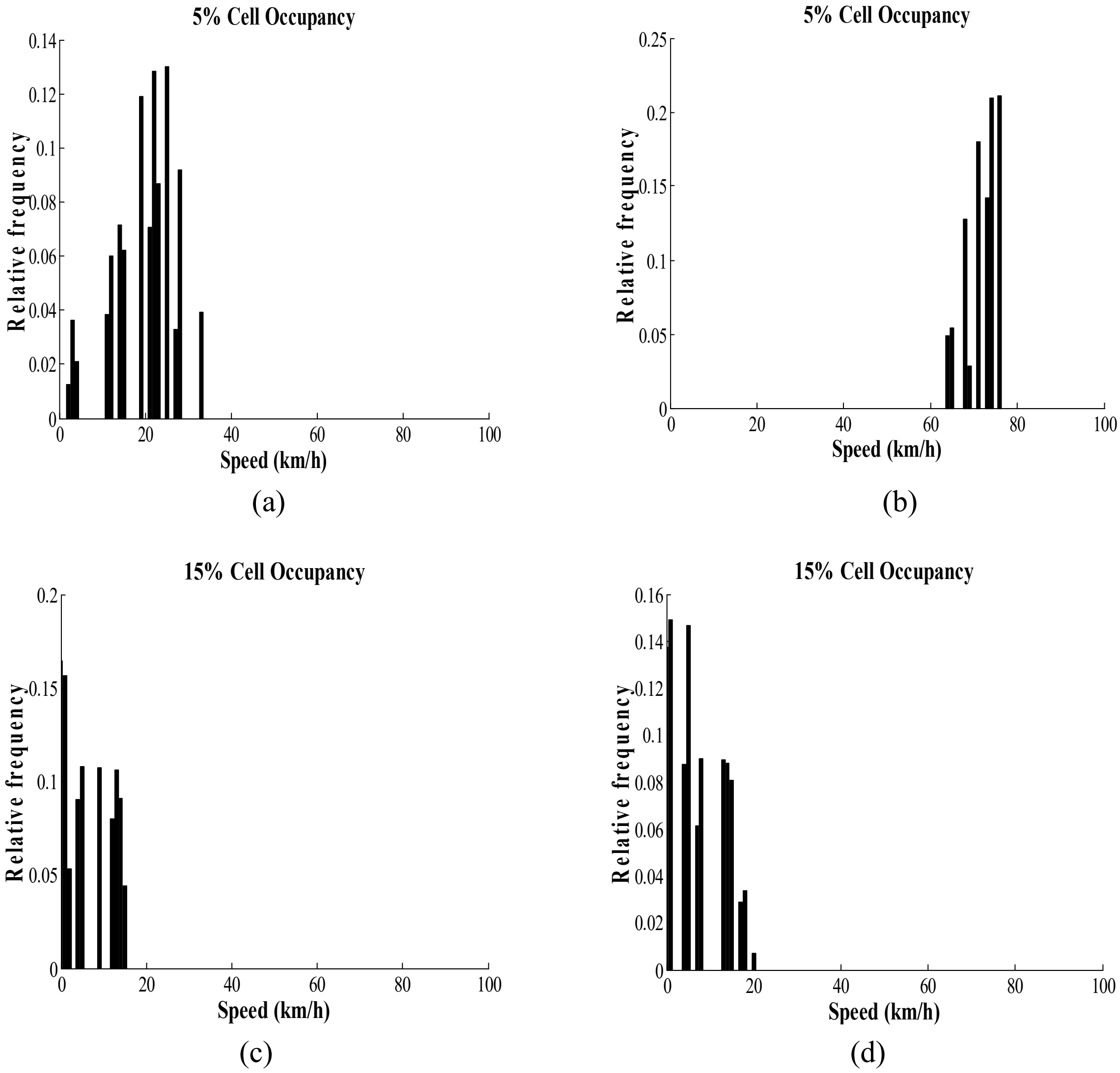

Figure 10: (a) Speed distribution for a two-lane road with one-lane blocked at the upstream for a cell occupancy of 5%
(b)Speed distribution for a two-lane road with one lane blocked at the downstream for a cell occupancy of 5%
(c) Speed distribution for a two-lane road with one-lane blocked at the upstream for a cell occupancy of 15%
(d) Speed distribution for a two-lane road with one-lane blocked at the downstream for a cell occupancy of 15%

As there is obstruction in the flow space, the number of vehicles crossing the upstream and moving to the downstream reduces drastically. Hence, there is a drop in capacity of the system. The capacity remaining for the system is presented in Table 1. The values obtained are comparable with the values suggested by Highway Capacity Manual 2010 (HCM 2010).

Table 1: Proportion of capacity available after the lane drop

Road Width (m)	*Proportion of capacity available*		
	3.5m blocked	*7m blocked*	*10.5m blocked*
7	0.38-0.42	0	-
10.5	0.48-0.53	0.17-0.21	0
14	0.60-0.63	0.30-0.32	0.16-0.19

Traffic flow under partially blocked road condition is presented in the present piece of work. This study is useful for developing countries (such as India) where the capacity values of a given facility for a specific Level of Service condition is provided in standard code of practice. These capacity values provided neglects the effect of partial blockage of a road facility which may occur due to construction activity, maintenance, accident and many more.Taking the help of HCM 2010, an attempt is made to develop a relation between a normal facility and a partially blocked facility.

5. Conclusion

In the present study, a cellular automata model has been developed to mimic traffic flow in work zone for only cars condition. Macroscopic and microscopic studies have been done for various road widths and blockage conditions. The spatial temporal profile shows that the static obstacles causes the vehicles to cluster at the upstream side of the obstruction. As a result, there is a long standing queue of vehicles which inhibits the relocation of vehicles from the downstream to the upstream end. This effect is more pronounced at higher cell occupancy. Speed distribution studies shows that at lower densities the effect of obstruction is less pronounced. Vehicles find it less difficult to move through the constricted region. Due to the jam condition induced by the static obstacle there is reduction in the number of vehicles passing the considered section. This results in the reduction of capacity. The capacity values are obtained by simulation are compared with the values suggested by Highway Capacity Manual 2010 (HCM 2010). The values seem to match very closely.

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